

Symbol Considerations for Bivariate Thematic Mapping

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Abstract

Bivariate thematic maps are powerful tools for understanding geographic phenomena, making visible spatial associations between them. But bivariate thematic maps are more visually complex than a univariate map, a source of frustration for both map creators and map readers. Despite a variety of visual solutions for bivariate mapping, there exists few 'best practices' for selecting or implementing an appropriate bivariate map type for a given scenario. This results in a need for empirical research examining the perceptual and functional differences among bivariate mapping solutions.

This research reports on a controlled experiment informed by the theory of selective attention, a concept describing the human capacity to tune out unwanted stimuli, and attend specifically to the information desired. The goal of this research was to examine if and how the perceptual characteristics of bivariate map types impact the ability of map readers to extract information from different bivariate map types.

55 participants completed a controlled experiment in which they had to answer close ended questions using bivariate maps. Accuracy and response time were recorded for each question. The experiment also opened with biographical questions to determine participant expertise and finished with a Likert-based survey to determine participant preference of the different map types.

The results of this experiment suggest that 1) despite longstanding hesitations regarding the utility of bivariate maps, participants were largely successful in extracting information from most if not all of the tested map types, and 2) the eight map types varied in terms of how intuitively participants were able to use the maps to answer the survey questions. While selective attention theory could explain some of these differences, the performance of the map types differed appreciably from similar studies that examined these map symbols in a more abstract, perception-focused setting. While the perceptual models of selective attention can still be useful in guiding map design, more work needs to be done in understanding the cognitive aspects and limitations to bivariate map reading.

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Chapter 1: Introduction

1.1 - Symbol Types in Thematic Mapping

Thematic mapping, or maps that indicate the variation of one or several statistical attributes across space, is a primary topic of interest for both the science and practice of Cartography. The topic has been the emphasis of much of the twentieth century scientific investigation within Cartography (e.g., McMaster & McMaster, 2002; Montello, 2002) and a topic of discussion within numerous Cartography textbooks (e.g., Dent, Torguson, & Hodler 2009; Fisher, 1982; Krygier & Wood, 2011; Robinson, 1995; Slocum et al., 2003; Tyner, 2010). Thematic maps are now a familiar means of visualizing information, visible in newspapers, magazines, journal articles, and elsewhere (Monmonier, 1999).

The thematic map literature details the considerations and best practices regarding thematic map design; subjects like the appropriate choice of color schemes (Brewer, 1989; Brewer et al., 1997; Brewer & Olson, 1997), the means of assigning data into classes (Brewer & Pickle, 2002; Jenks & Caspall, 1971), and algorithms for perceptual scaling of proportional symbols (Brewer & Campbell, 1998; Flannery, 1971). Each of these studies deals with particular design decisions within a particular kind of map selected for representing a particular kind of geographic phenomenon. Comparatively less has been said about one of the most fundamental

choices a map designer must make: which style of symbolization to select for their map. For instance, choropleth, graduated symbol, isopleth, and dot density are common solutions for univariate maps and all can be used to represent the same numerical, enumerated information (MacEachren & DiBiase, 1991). In the following text, the term **map types** will be used to refer to competing forms of symbol styling that may be employed to represent geographic information, with discussion limited to the context of symbol types supporting thematic mapping.

There is a small, yet important set of contributions regarding the selection of map types for univariate thematic mapping. MacEachren & DiBiase (1991) prescribe the use of different common thematic map types according to the kind of geographic phenomenon to be represented, with the phenomenon varying across two axes: (1) continuity (discrete versus continuous phenomena) and (2) abruptness (abrupt versus smooth phenomenon). Additional guidelines are offered according to the dimensionality of the represented geographic phenomenon (e.g., point, line, area, volume) and its level of measurement (nominal, ordinal, or numerical) (MacEachren, 1995).

Different map types also may vary according to their functional effectiveness for map reading (Robinson, 1995). For instance, in a study comparing different symbol types, MacEachren (1982a) found that isopleths are more effective than choropleths in providing the reader a sense of the larger spatial patterns in the

information (high level or 'General' map reading tasks, as will be fully described in **Section 2.3**). This necessitates research testing the comparative strengths and weaknesses of different map types, establishing which map type is best employed for common map use tasks in order to allow the map designer to make an informed solution for visualizing their information. The MacEachren comparative study also revealed nuances in such map type prescription; at least one map reading task was better supported by choropleth rather than isopleth maps (the recall of General spatial patterns in the information, but only when the maps had large numbers of classes). Thus, while there may be one overall 'winner', there likely are a variety of contextual constraints and influences that may promote one map type over the other given the user's task. Other comparative studies that exposed such nuances in map type selection include Johnson's (2008) user testing of different forms of cartograms and Nelson's (2000) bivariate map testing. Future work is still needed in testing map types to identify the subtle functional differences between map types.

1.2 – Bivariate and Multivariate Maps

Another major limitation in the existing thematic Cartography literature is that it primarily focuses on **univariate** maps, or cartographic representations that portray only one attribute of a geographic information set. Displaying two or more attributes (a **bivariate** and **multivariate** map, respectively) is a powerful way to convey information about associated geographic phenomena, but successfully designing

bivariate/multivariate maps is challenging due to the added density of information. To scope the problem, the following research will not address multivariate maps, and will consider bivariate maps only.

The functional purpose of a bivariate map is to show relationships among two geographic phenomena (Fisher, 1982; Tyner, 2010). Visualizing this geographic relationship frequently provides insight into understanding the mapped phenomena. When variables display an association over space, whether positive or negative, it suggests the phenomena have some influence on each other. Areas that do not reflect the larger-scale relationship between the variables can be identified; the fact that they 'buck the trend' suggests that there may be a confounding influence occurring in that region.

If enhanced understanding of a potentially related geographic phenomena is the benefit of bivariate mapping, comprehensibility is the cost. A bivariate map is, by its nature, more visually complex than a univariate map. **Visual complexity**, as defined by MacEachren (1982a; 1982b), describes the degree of intricacy created by the map elements. Visual complexity makes the map more difficult for the viewer to process mentally, and, if this complexity proves overwhelming, it can render the map valueless to the reader. Fisher (1982: 268) puts the bivariate map (or 'multi-subject' map, as he refers to it) in simple cost/benefit terms: there is "a limit beyond which the difficulty of comprehending two or more subjects exceeds the value of being able to relate them" and actually goes so far as to claim that multivariate mapping "is

desirable in very few circumstances." Similar sentiments are offered by Beard & Mackaness (1993) and McGranaghan (1993) in the context of uncertainty visualization (where the attribute and some measure of its uncertainty are represented together in a bivariate map). Importantly, several studies regarding bivariate maps have revealed a variation of effectiveness across the expertise of the map reader (e.g., Hope & Hunter, 2007; Kobus, Proctor, & Holste, 2001; Roth, 2009; see **Section 2.4** for more details). The issue of visual complexity within bivariate mapping is directly related to the concept of *selective attention*.

1.3 – Selective Attention and Bivariate Map Reading

Defined simply, ***selective attention*** is the ability of an observer to attend to one stimulus while ignoring the confounding influence of others. In the context of complex visual stimuli (such as a map), selective attention manifests as the ability to attend to specific ***visual variables*** while ignoring the others. Visual variables are the low-level graphic dimensions of an image (size, shape, color, etc.; see **Section 2.1.2** for a discussion of visual variables within Cartography). A univariate map requires the employment of only one visual variable for encoding data; for example, a graduated symbol map varies the size of the symbol while its color, shape, and so forth are kept constant. With a bivariate map, the presence of additional visual variables potentially creates interference; it becomes more challenging to retrieve individual attribute values from the symbols or to notice broad geographic patterns across the map. The concept of selective attention was developed by experimental

psychologists interested in how humans process images, and has been suggested as a useful theoretical framework for structuring comparative experiments on map types within Cartographic research (Shortridge, 1982).

Selective attention, however, is not purely an obfuscating influence on the bivariate map. The map designer can potentially capitalize on the way humans process images in order to better communicate some aspect of the geographic information. This is due to the phenomena of separability and integrality of visual dimensions (the additional properties of *configurality* and *asymmetry* will be discussed in **Section 2.2**). If the observer is able to attend to one visual variable without much interference from the other, the combination of those visual variables are considered **separable**. A common example of separable visual variables are value and shape (**Figure 1.1: left**); given an array of symbols that vary in both shape and value (lightness), an observer can distinguish swiftly between the different shapes, as well as light and dark symbols. **Integral** visual variable combinations, on the other hand, cannot be individually attended to when combined, instead forming an **emergent** or **gestalt dimension** (Carswell & Wickens, 1990). When given an array of rectangles that vary in height and width, it is challenging to separate the wide rectangles from the thin ones, and the tall ones from the short (**Figure 1.1: right**). Rather, we immediately perceive the combined influence of height and width, focusing on the emergent visual dimension of the symbol: its area.

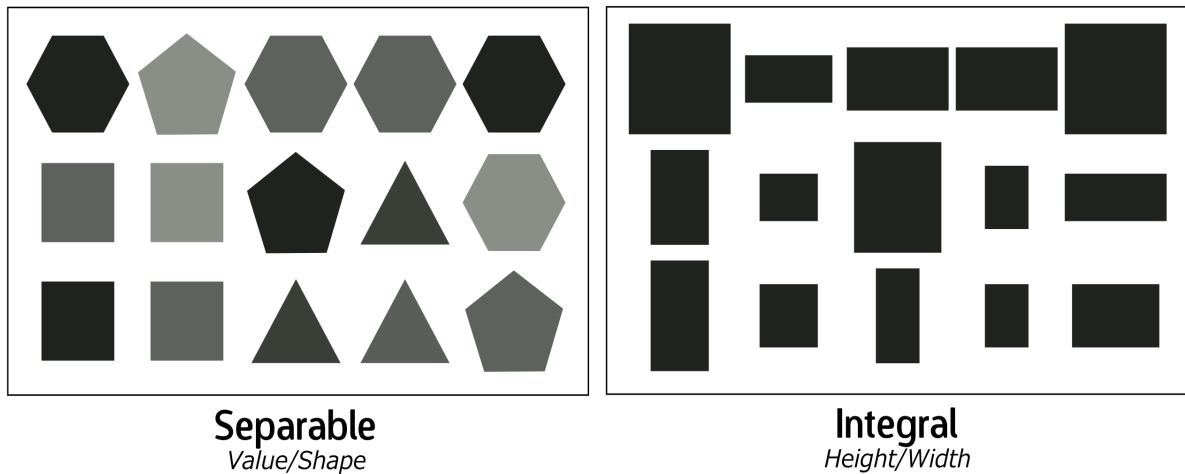


Figure 1.1: Integral versus separable visual combinations. Both series of symbols portray the same information, one using height/width and the other using value/shape. Note the cluster in the lower left corner is easier to identify when portrayed as four squares, rather than when portrayed as four rectangles of equal width. Contrarily, it is easier to pick out large rectangles, as opposed to dark hexagons.

In cartography, certain map reading tasks can theoretically benefit from keeping the visual variables separable, while others are accommodated by the emergent properties of integral combinations. Integrality, for instance, can be used to focus the reader's attention by highlighting or downplaying certain items on the map. This is the basis for many techniques of mapping uncertainty within information; the visual variables are employed such that highly certain map features stand out, while the less certain features do not (e.g., Leitner & Butterfield, 1993; Nelson & Edwards, 2001; Huffman, 2010). Separability between the symbols is preferable when, for instance, creating a bivariate map portraying two independent variables. The user can attend to each set of data with less visual interference (Shortridge, 1982).

1.4 – Problem Statement and Research Questions

The paucity of user-based, comparative testing of bivariate map types presents a significant problem for the thematic map designer; with little theoretical or empirical basis from which to draw, the designer cannot make an informed decision on which bivariate symbolization scheme to use. Selecting a map type for a bivariate map is especially challenging. Numerous bivariate map types are already in standard use, and new solutions continue to be proposed (e.g., Roth, Woodruff, & Johnson, 2010). Because of the variety of ways univariate map types can be 'combined' to form a bivariate map, there are many more design possibilities for bivariate mapping compared to univariate mapping. Given this, the need to understand the capabilities and limitations of these various bivariate map types becomes all the more pressing. As stated above, prior research on bivariate map symbols (e.g., Carswell & Wickens, 1990; Nelson, 1999; 2000) has suggested that choice of map type will facilitate (or complicate) different map reading tasks, but further research is necessary both to organize bivariate map types and explore their relative strengths and weaknesses.

Therefore, the goal of the research reported in the following manuscript is to provide additional insight into the effect that competing bivariate map types has on map reading. To accomplish this goal, a controlled experiment was conducted to derive empirically the strengths and weaknesses of different bivariate map types,

particularly considering the nuances of each bivariate symbol solution given the map reading task and map user context. Specifically, this research will seek answers to three interrelated research questions:

1. Do the selective attentive characteristics of bivariate map types (separable, integral, configural, asymmetrical) impact their general effectiveness or efficiency?
2. Does this efficiency and effectiveness vary across map reading task? As reviewed above, studies such as MacEachren (1982; 1995) and Nelson (1999; 2000) found that utility of the map type varies across different map reading tasks.
3. Does this efficiency and effectiveness vary across characteristics of the map reader? As reviewed above, several studies investigating bivariate maps that represent information and its uncertainty found that utility of a thematic map varies across the expertise of the map user.

The results of this research aid both in choosing appropriate symbol types for bivariate mapping, and for structuring future controlled experiments on bivariate mapping.

Chapter 2: Background

2.1 – Understanding Bivariate Map Types

2.1.1 Overview

Bivariate maps are often described as a combination of two univariate map symbols (Tyner, 2010). One example is a choropleth map with graduated symbols overlaid on top of it (***choropleth with graduated symbols***). Color is frequently applied to value-by-area cartograms, producing a map with the features of both a choropleth and cartogram (***shaded cartograms***). Given the numerous possible combinations of univariate maps, it is helpful to systematically catalog these combinations. Constructing such a taxonomy of bivariate map types provides a conceptual framework with which to organize existing bivariate map types, and potentially identify novel combinations of symbol types not in common use. Such a taxonomy also informs the empirical comparison of these different bivariate map types, in terms of the separability or integrality of their symbols, a factor increasingly recognized as an important component of their functionality (Nelson, 1999; 2000).

Textbook chapters on bivariate and multivariate thematic mapping are less systematic than, for example, their writing on univariate map types, map projections, or color schemes. A synthesis of six different thematic cartography textbooks (Dent,

Fisher	Robinson	Slocum et al	Dent, Torguson, & Hodler	Tyner	Krygier & Wood
Multiseries Graduated Symbol					
Choropleth with Graduated Symbols				Choropleth with Graduated Symbols	Choropleth with Graduated Symbols
Bar Graph					
Multiseries Dot Density			Multiseries Dot Density	Multiseries Dot Density	
	Graduated Pie Charts			Graduated Pie Charts	
		Bivariate Choropleth	Bivariate Choropleth	Bivariate Choropleth	
		Ray Glyph			
		Rectangle Map (Height/Width)			
			Shaded Cartogram		Shaded Cartogram
			Shaded Graduated Symbols		Shaded Graduated Symbols
					Isoline with Graduated Symbols

Table 2.1. Thematic Cartography texts and the bivariate map types discussed by each.

Torguson, & Hodler 2009; Fisher, 1982; Krygier & Wood, 2011; Robinson, 1995; Slocum et al., 2003; Tyner, 2010) and the bivariate map types they cover is provided in **Table 2.1**. Altogether, these sources identify eleven different bivariate map types, which represent only a small selection of map types identifiable from published maps or the cartographic literature. No text describes more than four different bivariate map types, and only two map types (bivariate choropleths and choropleth with overlaid graduated symbols) are considered by more than two sources.

Nelson (2000) offers what may be the closest approximation to a systematic taxonomy of bivariate map types, enumerating existing bivariate map types according to combinations of the visual variables (**Figure 2.1**). Most of Nelson's visual variable combinations are relatable directly to established bivariate map types: the squares that vary in size and value approximate a shaded cartogram or shaded proportional symbol; the height/width rectangle symbol also is an established bivariate map symbol (see MacEachren, 1995; Tyner, 2010).

Ultimately, however, Nelson's catalog of bivariate symbols does not serve as a complete taxonomy of bivariate map types, as it is not exhaustive (for example, there's no typeface/hue combination), although it was not intended to be exhaustive.

The concept of organizing bivariate symbols based on their constituent visual variables is a useful one, however, as visual variables serve as a fundamental graphic constituent of all map types (MacEachren, 1995) and information graphics

Figure 2.1. A selection of bivariate symbols and their constituent visual variables. The symbols are labeled with their nearest cartographic equivalent; italicized names denote map types in common use. Modified from Nelson (2000, p. 64)

more broadly (Bertin, 1967|1983).

This writing will expand upon the idea of organizing bivariate map types as combinations of visual variables by considering another fundamental constituent of a map symbol, its **dimensionality** (point, line, polygon, etc.) (Bertin, 1967|1983; Stefan et al., 2007). Dimensionality, in combination with visual variables, can be used to identify every bivariate map symbol that can be constructed (**Figure 2.2**). The following two subsections will discuss the extent of visual variables and dimensionalities, constructing an example taxonomy of map symbols based around these two fundamental constituents.

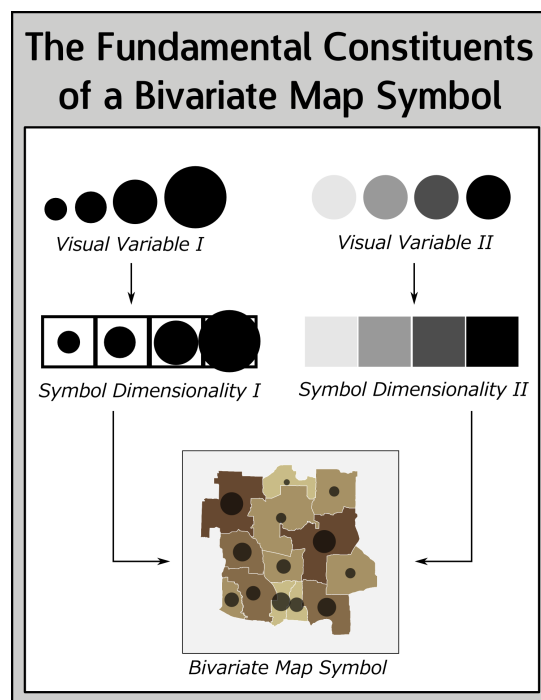


Figure 2.2 – The constituent parts of all bivariate map symbols: two visual variables, paired with two symbol dimensionalities.

2.1.2 – The Visual Variables

There is no broadly accepted list of visual variables, nor is it broadly presumed that such a list can currently be achieved, as advances in display technologies and novelties in graphic solutions continue to expose more visual variables. Tyner (2010) synthesizes visual variable taxonomies from seven Cartographic authors, reproduced in **Table 2.2**. Between the authors, a total of 13 distinct visual variables are proposed, with only four of those (size, shape, hue, and value) shared among all seven taxonomies. This writing will rely on its own synthesis of published visual variables, however this collection is closely influenced by MacEachren (1995). A visual assembly of these visual variables is provided in

Figure 2.3.

Dent et al	Kraak & Ormeling	Krygier & Wood	MacEachren	Monmonier	Slocum et al	Tyner
Size	Size	Size	Size	Size	Size	Size
Shape	Shape	Shape	Shape	Shape	Shape	Shape
Hue	Hue	Hue	Hue	Hue	Hue	Hue
Lightness/Value	Value	Value	Value	Value	Lightness	Lightness
Texture	Texture	Texture	Texture	Texture		Texture
Orientation	Orientation		Orientation	Orientation	Orientation	Orientation
Saturation/Intensity		Intensity	Saturation		Saturation	Saturation
Arrangement			Arrangement		Arrangement	Arrangement (pattern)
Focus			Crispness			
					Spacing	
					Perspective Height	
			Resolution			
			Transparency			

Table 2.2: A comparison of visual variable taxonomies. Table modified from Tyner (2010).

The visual variables selected for inclusion in **Figure 2.3** represent the contributions of several authors. The first set of visual variables were proposed by Bertin (1967| 1983; the terminology translated directly to 'retinal variables'). Bertin's original six retinal variables consisted of *size*, *value*, *grain* (texture or pattern to other authors), *color*, *orientation*, and *shape*. Morrison (1974) adapted Bertin's variables, subdividing color into its constituent parts: *hue* (the color itself; red, blue, green, and so on), *value* (darkness), and *saturation* (color intensity or purity). Caivano (1990) expanded upon the concept of texture, describing three constituents to any pattern fill: *directionality* (the apparent direction of the pattern, such as horizontal vs. vertical), *pattern size* (the scaling of the pattern), and *density* (the number of discrete pattern elements per unit area). MacEachren (1992), capitalizing on technological advancements in graphical displays, proposed *crispness* (blurriness), *resolution* (generalization), and *transparency* as visual variables appropriate for cartographic use. Presumed to be effective for representing uncertainty, MacEachren called the trio 'focus'.

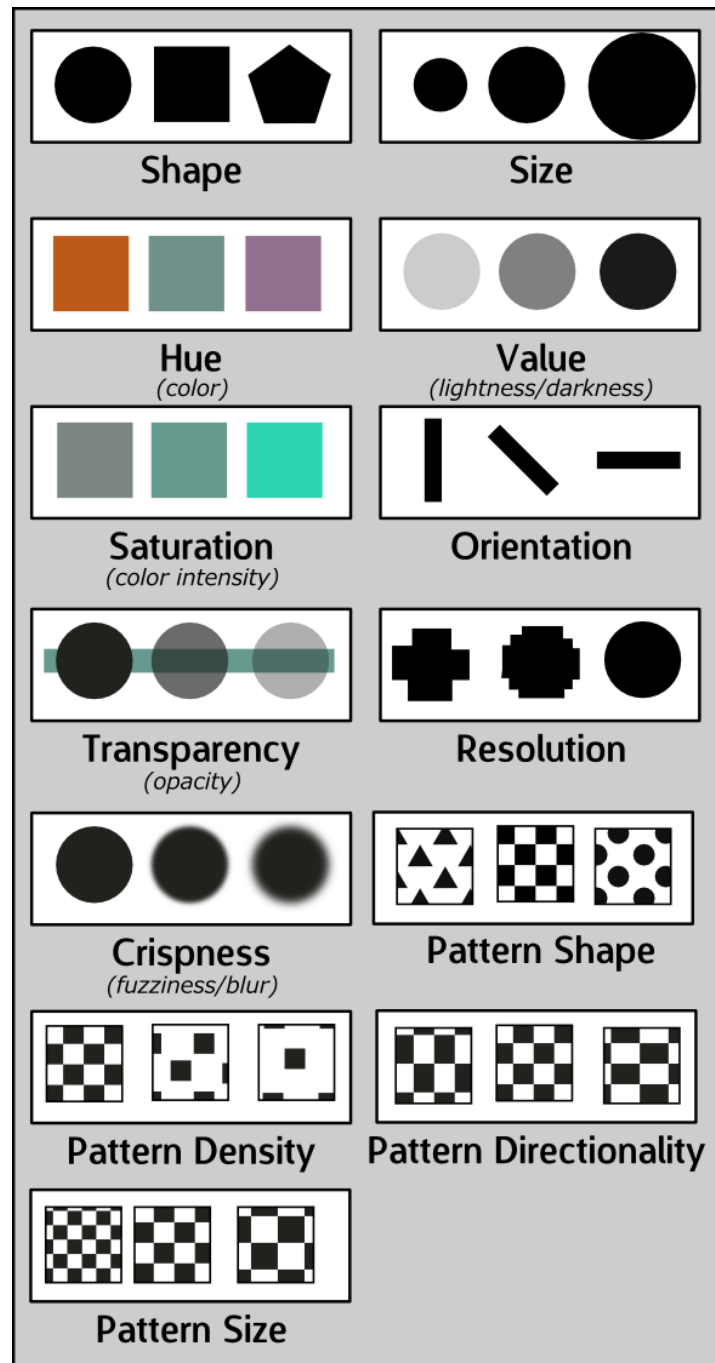


Figure 2.3 – A collection of visual variables.
 Synthesized from Bertin (1967|1983), Morrison (1974),
 Caivano (1990), and MacEachren (1992).

2.1.3 – Synthesizing Combinations of Visual Variables and Symbol Dimensionality

Functional bivariate map designs are constrained to the visual variables and symbol dimensionalities are appropriate for depicting the given sets of data. This section constructs an example of an exhaustive solution space of bivariate map solutions, scoped to the common bivariate mapping problem of depicting numerical information enumerated to areal units.

Visual variables have varying appropriateness for depicting certain kinds of information. Bertin (1967|1983), for instance, distinguishes ordered from non-ordered variables: a non-orderable variable, like shape, is not precognitively 'ranked' by the reader; a pentagon is not understood as representing more than a triangle. Ergo, shape is an inappropriate variable for representing ordinal or numerical level data, as opposed to ordered variables such as value or size. A thorough categorization of visual variables and their congruency with depicting different forms of information is offered by MacEachren (1995), who designated twelve visual variables as 'good', 'marginally effective', or 'poor' at depicting numerical, ordinal, or nominal level data (**Table 2.3**).

For depicting numerical data, MacEachren (1995) discounts the use of non-orderable visual variables (arrangement and shape), as well as the focus variables (crispness, resolution, and transparency). The latter three variables are suggested as more suited for representing ordinal measures of uncertainty, rather than a

	Numerical	Ordinal	Nominal
<i>Size</i>			
<i>Crispness</i>			
<i>Resolution</i>			
<i>Transparency</i>			
<i>Color Value</i>			
<i>Color Saturation</i>			
<i>Color Hue</i>			
<i>Texture</i>			
<i>Orientation</i>			
<i>Arrangement</i>			
<i>Shape</i>			

Table 2.3 - Visual variable functionality as proposed by MacEachren (1995). Darkness indicates that visual variable is suitable for depicting that level of data.

numeric independent variable. In the case of transparency, though, Roth, Woodruff, & Johnson (2010) create convincing examples using transparency to encode a numerical, equalizing variable (population). Thus, among the original visual variables identified in **Figure 2.3**, the ones conducive to representing numerical data are size (divided into height and width), transparency, value, saturation, hue, orientation, and the various dimensions of pattern fills (size, hue, saturation, value, density, and orientation).

As previously mentioned, describing existing bivariate map types based on these visual variables requires a second consideration: the **dimensionality** of the symbol. That is, whether the map symbol is a point (0D), line (1D), or polygon (2D)

(Tyner, 2010). Volumetric (3D) map symbols are possible, but will not be considered in this study. Two bivariate map types might employ the same pair of visual variables in their map symbols, but differ according to the dimensionality of the symbols. For instance, either a shaded cartogram or choropleth with overlaid graduated symbols would represent a size/color combination, however a shaded cartogram employs size and color within a polygonal symbol, and a choropleth/graduated symbol map uses one polygonal symbol (the choropleth) and one point symbol (the graduated symbols). The dimensionality of a symbol does not necessarily match the dimensionality of the geographic phenomena (Tyner 2010); point symbols, placed at the center of the areal unit, are commonly used to represent an attribute value for that area, as with the graduated symbol example. Line symbols are not as appropriate for depicting areal features (ibid), leaving only point and polygon symbols appropriate for depicting information enumerated to a two dimensional areal unit. In a strictly bivariate application, that creates three possible combinations of dimensionality for the symbol (**Figure 2.4**).

- Polygon/Polygon:** both visual variables are applied to an area symbol (generally the enumerated areal units). An example is a shaded cartogram, where the political units (states, countries, etc.) vary by size and color.

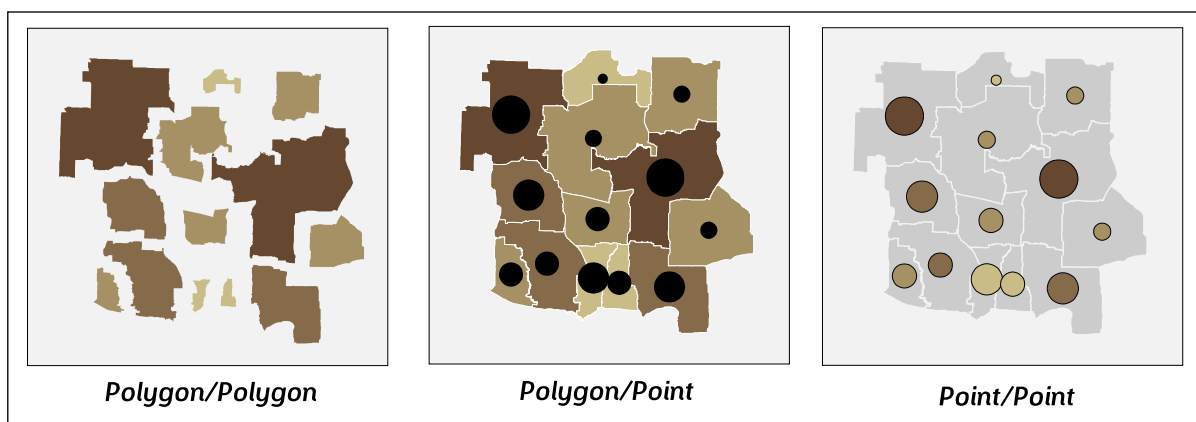


Figure 2.4: All three maps portray the same information, using the same visual variables (size/value), but with different combinations of symbol dimensionality.

- Polygon/Point:** a point symbol is superimposed onto a polygon symbol. The point and polygon symbols each vary by one visual variable. One example is a choropleth map (polygon symbol, varies by color) with superimposed graduated circles (point symbol, varies by size).

- Point/Point:** a bivariate glyph. Both visual variables are applied to a point symbol, with no statistical information encoded into the 'basemap'. A common example would be a rectangle map, where the point symbols are rectangles whose height and width each represent one variable.

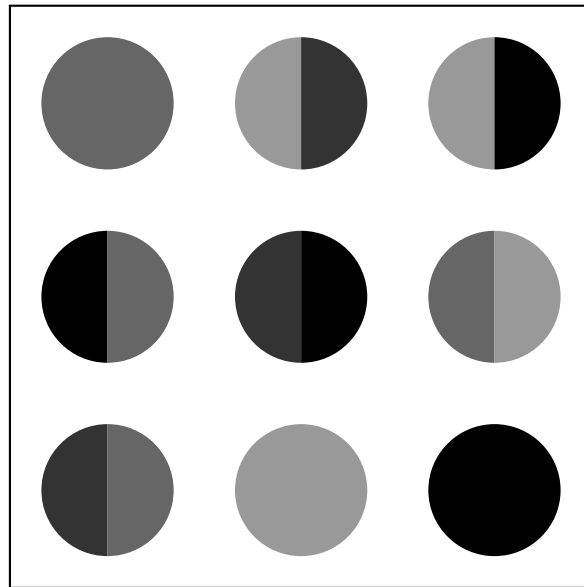
Tables 2.4, 2.7, and 2.8 treat each of these dimensionality pairings individually, in combination with the visual variables outlined above, summarize every basic and tenable way to produce a bivariate map of enumerated, numerical information.

2.3 – The Role of Selective Attention

2.2.1 – Conditions of Selectivity

As mentioned in **Section 1.3**, the separability or integrality of a bivariate map type strongly influences the way the map reader mentally processes it, and therefore should influence its functionality in a map use setting. The selective attentive characteristics of a symbol (hereafter referred to as its **selectivity**) are dependent on the visual variables used in the symbol's construction (MacEachren, 1995; Nelson, 2000; Shortridge, 1982; Carswell & Wickens, 1990). One major challenge to selectivity is that the distinction introduced in **Section 1.3** between 'integral' and 'separable' visual variable pairs is likely a false dichotomy. Multiple authors have, instead, suggested integrality and separability as opposite ends of a continuum (Shortridge, 1982). Between these antipodes are visual combinations that exhibit certain qualities of both integrality and separability. MacEachren (1995) and Carswell & Wickens (1990) describe this intermediate level along the selectivity continuum as **configural** combinations. One illustrative example of configurality (via Nelson, 2000) is a circular symbol divided in half, with both halves varying in terms of their value (a *value-value* combination; **Figure 2.5**). It is possible to attend to just the left or right halves of the circle (separable), but there are also emergent features and dimensions that arise via the combination of the two halves (integral), such as the overall darkness/lightness of the symbol or the contrast between the two halves. In

practice, configural combinations tend to arise when a symbol employs the same visual variable twice, such as value/value or size/size (ibid).



***Figure 2.5:** A configural combination (value/value). Contrast between the two circle halves, as well as the overall darkness of the circle, are emergent features.*

The fourth condition of selectivity is **asymmetrical**, which, like configural, creates emergent visual dimensions characteristic of integral selectivity, while retaining some degree of separability (Carswell & Wickens, 1990; Nelson, 2000). The interference present in asymmetrical combinations is conditional: one visual variable may be difficult to parse while the other can be attended to with relatively little inhibition.

Asymmetrical combinations tend to arise when one constituent variable serves as a stronger visual cue than the other. The combination of numerosness

(pattern density) and size is an example (**Figure 2.6**). The emergent dimension is the relative area marked by the dots (i.e., the 'ink coverage'); this increases with both density and the size of the marks, creating redundancy when these two variables are in agreement. When they are not in agreement, the emergent dimension (coverage) can be equivalent between two different symbols, but based on different influences (either high density and low size, or large size and low density).

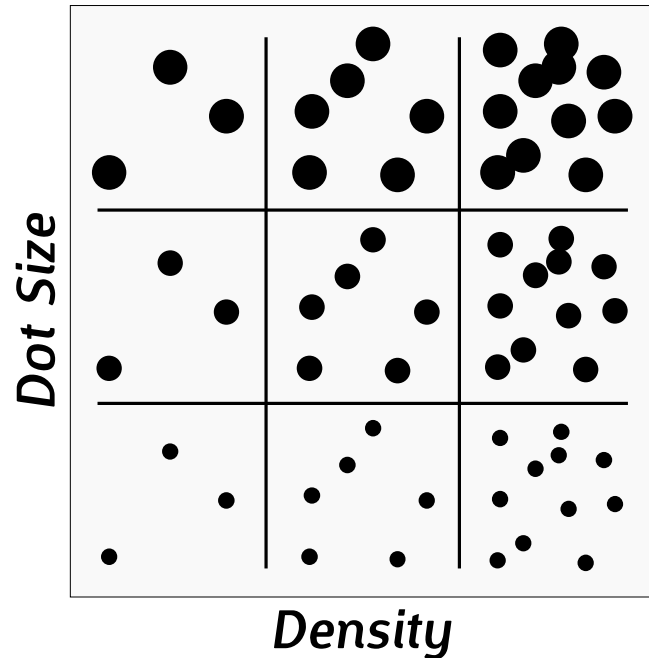


Figure 2.6: *Size/Density is an asymmetrical combination.*

2.2.2 – Selectivity of Map Types

Selective Attention research does not always examine symbols comparable to those familiar to information designers; nonetheless, several examinations have

determined the selectivities of visual combinations used in thematic cartography.

Table 2.8 presents all visual variable combinations (based on the catalog of visual variables presented in **Section 2.1.2**), and their selectivity, derived from the synthesis of Shortridge (1982), Nelson (1999; 2000), and Carswell & Wickens (1990).

Width	C									
Height	I	C								
Blur	S	S	C							
Resolution	-	-	-	C						
Transparency	S	S	-	S	C					
Value	S*	S*	S	S	S	C				
Saturation	S	S	S	S	S	I	C			
Hue	S*	S*	S	S	S	A	I	C		
Orientation	-	-	S	-	S	S	S	S	C	
Shape	-	-	S	-	S	S	S	S	-	
Fill Size	-	-	S	-	S	S	S	S	-	
Fill Shape	-	-	S	-	S	S	S	A	-	
Fill Hue	S	S	S	S	I	-	-	-	S	
Fill Value	S	S	S	S	I	-	-	-	S	
Fill Saturation	S	S	S	S	I	-	-	-	S	
Fill Density	S	S	-	S	S	-	S	S	S	
Fill Orientation	S	S	S	S	S	-	S	S	-	
Fill Directionality	-	-	S	S	S	-	S	S	-	
	Width	Height	Blur	Resolution	Transparency	Value	Saturation	Hue	Orientation	

Shape	C									
Fill Size	-	C								
Fill Shape	-	S	C							
Fill Hue	S	S	S	C						
Fill Value	S	S	S	I	C					
Fill Saturation	S	S	S	I	I	C				
Fill Density	S	A	S	S	S	S	C			
Fill Orientation	-	-	-	S	S	S	S	C		
Fill Directionality	-	-	I	S	S	S	-	I	C	
	Shape	Fill Size	Fill Shape	Fill Hue	Fill Value	Fill Saturation	Fill Density	Fill Orientation	Fill Directionality	

Table 2.8 – Combinations of visual variables and their attentive characteristics (table divided for legibility at print size). Selectivities are Separable (S), Integral (I), Configural (C), Asymmetrical (A), or unknown (-). Items in **bold** have been established by empirical study; greyed items are estimations by the author. * Note: determinations of value/width, value/height, hue/width and hue/height are based upon studies of hue/size and value/size.

Configural combinations, as understood by the literature, tend to arise from homogenous pairings of visual variables (Nelson, 2000). For **Table 2.8**, all homogenous pairings are presumed to be configural (see **Section 2.2.1**). No

combinations, other than ones identified in the literature, were presumed to be asymmetrical; the presence of this characteristic is the most difficult to predict, as the perceptual properties of asymmetry are nuanced compared to the others.

A large proportion of combinations in **Table 2.8** do not have established condition of selectivity in the literature. The unclassified combinations generally involve MachEachren's (1992) 'focus' variables (blur, transparency, and resolution) and the pattern fill variables described by Morrison (1974) and Caivano (1990). These variables were derived from a Cartographic background, and therefore would be unlikely to be seen in studies performed by Experimental Psychologists. The large number of gaps in the table suggest there is continued work to be done in reconciling the research fields of Cartography and Selective Attention.

2.2.3 – Selective Attention and Encoding Data

In a bivariate map symbol, the two attributes are encoded by the two visual variables used to construct the symbol. Each combination of visual variables also creates two emergent visual dimensions, which are visual variables unto themselves. A combination of height/width, for instance, creates two emergent visual dimensions: area and directionality (**Figure 2.7**).

These gestalt visual variables encode information in the same way the original visual variables do. One encodes a positive association between the data (i.e., map features where both attributes are low vs features where both attributes

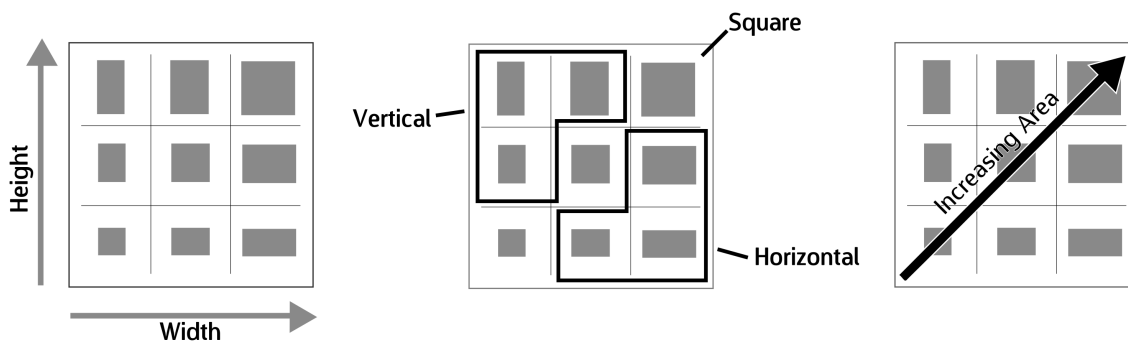


Figure 2.7: *The emergent dimensions of a height/width combination.*

are high) while the other gestalt dimension encodes negative association (features where one data variable is higher than the other, or vice versa). Given a matrix of symbols (as would be seen in a two-axis map legend), these emergent visual variables can be found along the orthogonal axes (**Figure 2.8**). (From herein, these orthogonal information axes will be referred to as the **Plus(+)** and **Minus(-)** axes, and the original information axes as the **X** and **Y** axes). Returning to the example in **Figure 2.7**, the Plus(+) axis in this symbol set will be encoded by the area of the symbol, and the Minus(-) axis will be encoded by the directionality of the symbol (ranging from horizontal to vertical, with perfect squares existing in between).

A variety of potential emergent visual dimensions exist when visual variables are combined (see **Figure 2.9**). These emergent dimensions appear to exhibit varying strengths as a visual cue. Within a bivariate symbol, the relative strength of the four visual dimensions (X, Y, and the two emergent dimensions) appear to determine whether the symbol will be separable, integral, configural or asymmetrical. Integrality, for instance, occurs when an emergent dimension (generally the axis of

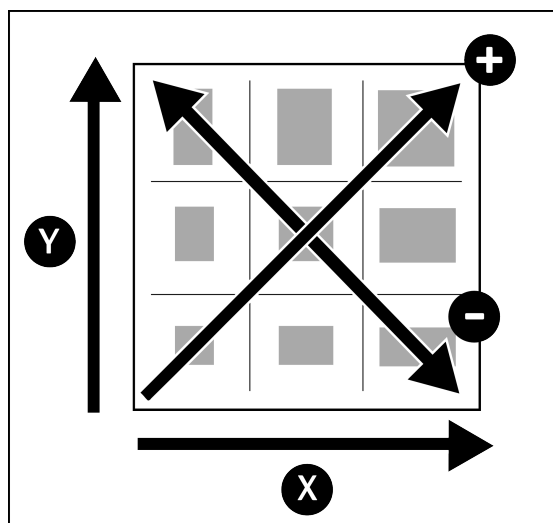


Figure 2.8: The orthogonal information axes in a bivariate symbol matrix (bivariate map legend).

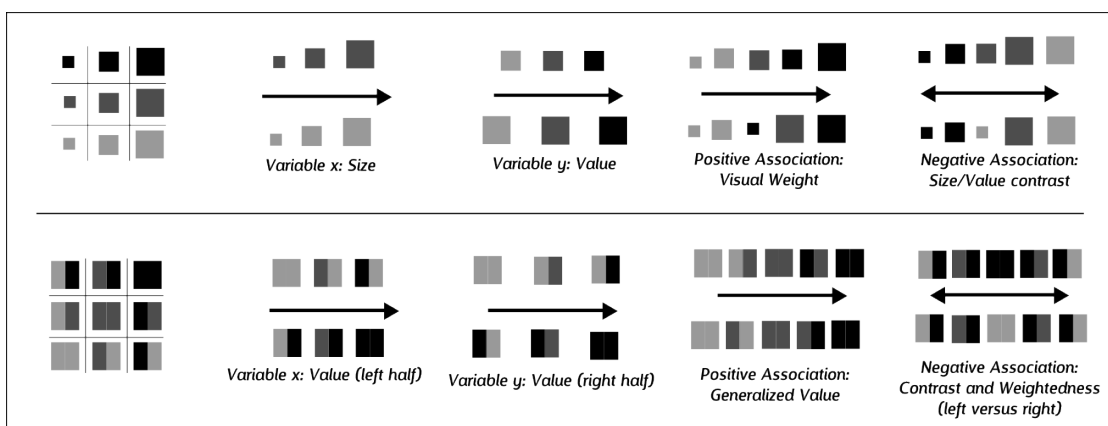


Figure 2.9: An example of two bivariate symbols and their emergent dimensions. The emergent dimensions are encoded along the orthogonal (Plus[+], Minus[-]) axes.

positive association) provides such a powerful visual cue that it interferes with the viewer's ability to attend to its constituent visual variables on the X and Y axes. A summation of the four categories of selectivity, and the hypothesized degree to

which they support the parsing of information along their four visual axes, is provided in **Figure 2.10**.

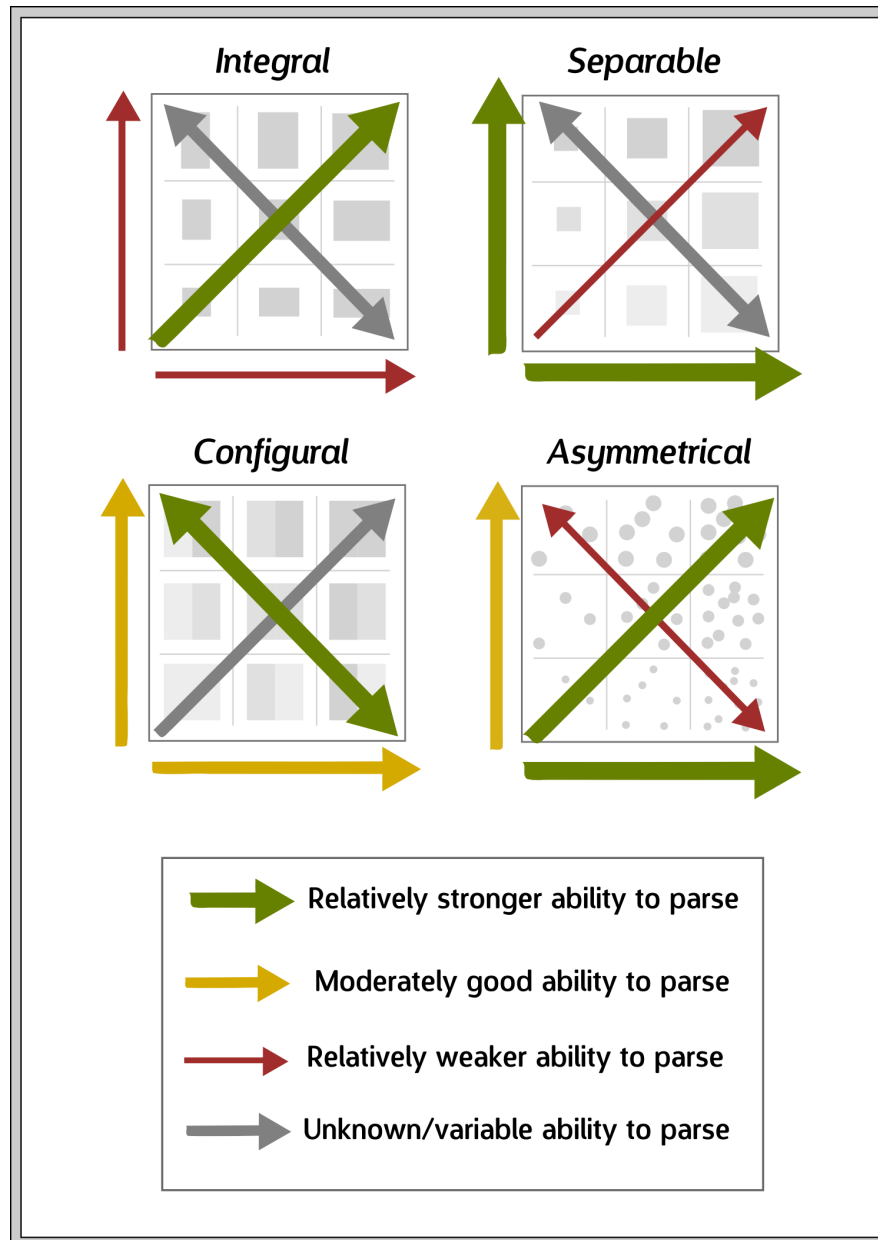


Figure 2.10: The four conditions of selectivity, based on the relative strength of their four visual dimensions. Note that there are multiple means of achieving asymmetry: the stronger visual cue may alternatively be found on the Y axis.

2.3 – Tasks in Bivariate Map Reading

At its most basic level, a thematic map facilitates the extraction and analysis of information by the map reader. Every map supports a variety of activities the reader can engage to acquire information from the map; the term ***map reading task*** is used to describe the single example of such activity. The design of a map, including its symbol design, supports different map reading tasks to different degrees. Often, the cartographer must sacrifice the efficiency of one task in the interest of another. One such example is the functional difference between classed and unclassed choropleth maps (Gale & Halperin, 1982). A classed choropleth map divides the information values into ranges, with each member of a class receiving identical shading; an unclassed choropleth map employs a color ramp such that each enumeration unit has its own unique color, according to its unique attribute value. Classed choropleths are believed to better support the extraction of individual attribute values, as well as better support the comparison of values between two noncontiguous areal units; unclassed choropleths have been recommended when the overall distribution of the attributes is more important than the values for individual areas (ibid), or when choropleth maps are sequenced within an animation (Harrower, 2007).

What map use tasks are relevant to bivariate mapping, and how are they influenced by symbol design? Nelson (1999; 2000) considers a tradeoff between separable and integral bivariate symbols. Separable symbol sets enhance the ability

of viewers to attend to the individual variables, while integral ones enhance their ability to visualize the relationship between the variables. Nelson's methodology is derived from the **speeded classification** studies of selective attention, modified for cartographic applications. Speeded classification is an empirical research methodology employed by researchers in Experimental Psychology, wherein a participant must, as swiftly as possible, match a given symbol to another within a small array of symbols. Within speeded classification, there are four essential tasks (illustrated in **Figure 2.11**): **baseline tasks** (viewer must attend to one visual variable, with the other visual variables held constant), **filtering tasks** (viewer must attend to one visual variable while ignoring the confounding influence of another), **redundancy tasks** (there is variation in both visual variables, and the viewer can attend to *either one*), and **condensation tasks** (there is variation in both visual variables, and the viewer must attend to *both of them*).

Although these four tasks effectively cover the breadth of potential interferences possible with bivariate symbols, they do not effectively reach the breadth of possible activities within reading a bivariate map (or thematic map in general). Bivariate maps often involve a greater number of symbols than a speeded classification task, and the viewer often must **visually aggregate** a large number of symbols in order to understand large-scale geographic patterns in the mapped

phenomenon. The reader must be able to relate the visual dimensions of the symbol to attribute values in the mapped information, whereas in speeded classification the viewer need only to determine whether two given symbols are

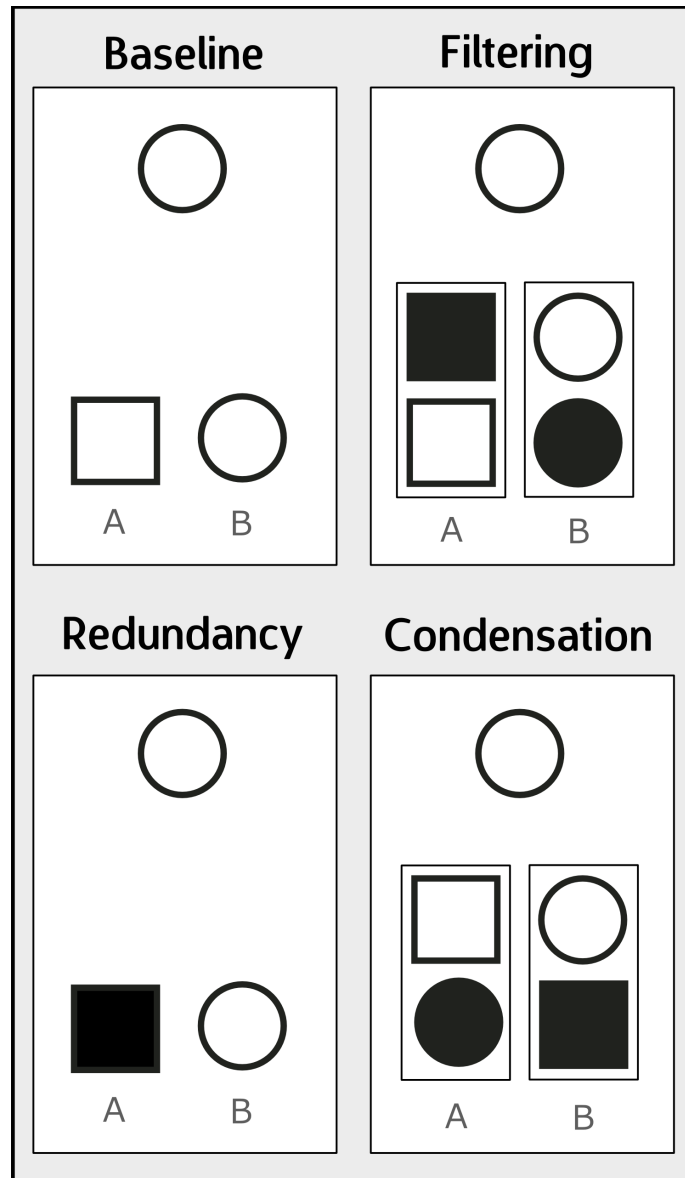


Figure 2.11: The four tasks within speeded classification. Modified from Carswell & Wickens (1990) and Nelson (2000).

identical. Maps also must place the symbols in the context of both a map and a legend, providing a context that can influence how viewers will interpret the symbols.

Multiple authors have attempted to provide a taxonomy of user tasks specific to the context of thematic cartography. Roth (2011) offers a synthesis of these objective-based taxonomies. Some of these taxonomies (e.g., Zhou & Feiner, 1998; Crampton, 2002; Amar, Eagan, & Stasko, 2005; Yi et al., 2007) include tasks specific to interactive mapping or exploratory data analysis in general, such as running calculations from the data, or visual reexpression of the information, and are therefore ill-suited for this research, which focuses on static maps. Regardless, the most commonly offered taxonomy (shared by Wehrend & Lewis, 1990; Blok et al., 1999; and Andrienko, Andrienko, & Gatalsky, 2003) is also the simplest, containing only two objectives: **identify** (extract information about single map object) and **compare** (comparing & contrasting two or more map objects). In the setting of a bivariate map, identify and compare can be applied along all four information axes (X, Y, Plus[+], Minus[-]) to accomplish what would otherwise be more sophisticated goals. Identifying a map object along the Plus(+) or Minus(-) axes, for instance, provides a 'shortcut' to describing the association between the two data variables within that map object.

Map reading tasks are capable of being performed at different visual levels, a concept first articulated by Bertin (1967|1983). Bertin distinguished between **elementary** tasks (those which consider only individual objects in the graphic),

intermediate (also known as **general**) tasks (those which consider clusters of objects containing several graphic objects), and **global** tasks (those which the viewer attends to the overall distribution of graphic objects). In a bivariate map, an elementary task would involve attending to single map features (a single areal unit or a single point symbol). General and global level tasks would involve examining regional and global level patterns. Although Selective Attention research has examined how humans are capable of seeing multiple graphical elements as a single coherent group (Pomerantz & Schwaartzberg, 1975), the distinction between elementary and general level tasks appears unique to information visualization.

2.4 – Role of Expertise in Map Reading

Audience is an important consideration in map design; each viewer's understanding of the map will be filtered by their particular knowledge base, attitude, expectations, and cognitive faculties (MacEachren, 1995). A functional map is designed to accommodate the abilities and needs of its expected audience. Bivariate maps, being a broad category of thematic maps, have a highly variable audience. Their context ranges from sophisticated exploratory mapping to communicating information to the general public.

It can be expected that users with different levels of expertise in map reading and spatial analysis will have varying capacity to process the visual of a bivariate map. **Expertise** is defined as the knowledge and skills learned by the user to

enhance and append one's innate abilities (Roth, 2011). In Selective Attention research, it is recognized that the way a viewer processes an image shifts once they are familiar with the classification task (Shortridge, 1982). An experienced map reader may process a map both faster and in a distinctly different way than a member of the general public, meaning that certain map types may be more or less appropriate for usage by a given audience.

User testing in cartographic research does not frequently consider the potential influences of expertise. The participants' abilities are implicitly presumed to be reflective of all map readers. Several cartographic studies have examined the influence of expertise in their investigations, however. Evans (1997) tested subjects' preferences between a variety of maps showing reliability information for a land cover map. Participants were separated into either the 'novice' or 'expert' category. The former group was composed of undergraduate students who had taken a Cartography-related course. The latter were students and professors who had some training in either GIS, Remote Sensing, or Cartography. Experts spent a longer time looking at the maps before making a decision, but otherwise the two group's performance was similar.

Cliburn et al. (2002) and Slocum et al (2004) incorporate the role of expertise in qualitative studies of dynamic displays. Cliburn et al. solicited feedback from hydrology experts, usability engineers, and regional policymakers on an a hydrological visualization. Slocum & Sluter ran interviews and focus groups with

non-geography students ('novices'), geography majors and grad students, and domain experts to obtain their impressions of an animated mapping software. Both studies were able to identify and articulate each group's varying expectations and critiques of the map display.

Hope & Hunter (2007) and Roth (2009), similar to Evans (1997), investigated participants' decision-making when given maps providing uncertainty information, and divided participants into cohorts based on expertise. Hope & Hunter's subjects self-classified themselves as either 'novice', 'some experience', or 'experienced', based upon the amount of time they had experience with GIS (less than six months, between six months and two years, and more than two years, respectively). Roth used several self-reported measures (work experience, education/training, and personal experience) to classify participants into three classes of expertise: novices, map use experts, and domain experts (in this study, the domain was floodplain analysis). The results showed that experts (both domain and map-use) were confident in reading the map and arriving at decisions based on its information (they reported low feeling of difficulty and intermediate-to-high confidence in their responses), however the map use experts were not significantly better than novices in the accuracy of their assessment.

Chapter 3: Methods

A controlled experiment was administered to assess the variation in performance across various bivariate map design solutions. The purpose of the study was to examine empirically how different map types and different conditions of selectivity support different bivariate map reading tasks, and if this level of support varies according differences in the expertise of the map reader. Conceptually, the experiment was designed to reconcile, at least in part, the methods employed in speeded classification and the methods employed in cartographic performance testing. This involved asking participants to perform a selection of map reading tasks across eight different bivariate map types, recording their accuracy and response time to each question. The survey also included questions on the users' personal preferences of the different map types.

3.1 Participants

A total of 55 participants participated in the controlled experiment. The majority of participants were recruited from the Geography Department of the University of Wisconsin – Madison, although the study was open to any interested participants. Participants were recruited purposefully to represent a range of experience and knowledge of cartography and spatial analysis. Recruiting methods included in-person advertisements given at the start of Geography lectures and

targeted e-mails to Geography-related listservs. The study participants were offered 5 USD as compensation for their time. Funding for the study was provided by the Trewartha Graduate Research Award.

Biographic information about the participants was collected at the start of the experiment, designed to assess the participants' expertise across the three dimensions of expertise used by Roth (2009): work experience, education & training, and personal experience. A summary of the participants' biographical information is provided in **Table 3.1**.

Measure	Question	# of Respondents (Yes)	# of Respondents (No)
Education	"Have you already completed a course on map design, map use, or GIS?"	35	20
Experience	"Does your current job or previous job require you to design maps, use maps, or use GIS in any way? "	30	25
Familiarity	"On a scale of 1 to 7, please rate your familiarity with maps:"	(Mean response: 4.87)	

Table 3.1: Participant summary table.

3.2 Materials

Eight common bivariate map types, drawn from **Tables 2.5-2.7**, were designed for inclusion in the experiment. Bivariate map types were selected to include two maps types for each condition of selectivity (separable, integral, configural, and asymmetrical). These map types were selected based on the following criteria: 1) if possible, there was existing research establishing the map type's selectivity (refer to **Table 2.8**), 2) the map type was particularly representative of the perceptual features associated with its selectivity, and 3) the map type is

commonly used in cartographic practice. The eight map types selected were **shaded cartogram** (size/value, separable), **choropleth with graduated symbols** (size/value, separable), **rectangle map** (height/width, integral), **bivariate choropleth** (value/hue, integral), **bar chart** (height/height, configural), **spoke glyph** (orientation/orientation, configural), **value-by-alpha** (hue/transparency, asymmetrical), and **shaded texture** (value/pattern density, asymmetrical) (Figure 3.1).

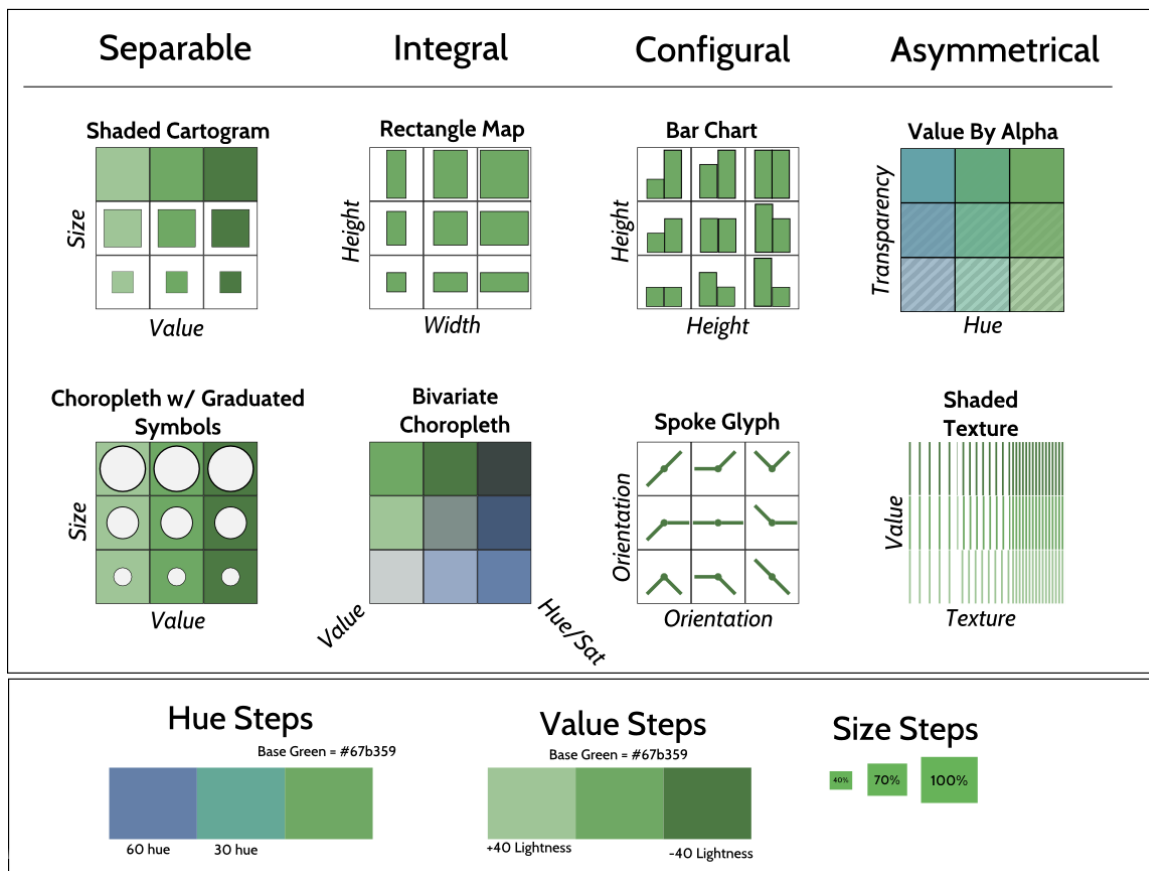


Figure 3.1. The eight map types included in the experiment.

The map legends were designed following several principles, in order to control for possible confounding factors. All maps employed the same green hue, excepting those requiring hue as a visual variable. These maps employed a green-blue color ramp, presumed to remain legible under most forms of color vision deficiency (Gardner, 2005; Jenny & Kelso, 2007). The various legend designs employed the same scaling of size, value, and hue when needed.

The design for the bivariate choropleth uses a modified construction from the other legend designs: the visual variables are applied across the orthogonal information axes, and it employs small variations in saturation as well as hue. This was done to align the legend design with established recommendations for the design of bivariate choropleth legends (Trumbo, 1981; Dunn 1989). The design for the value-by-alpha legend included a pattern fill to allow for a better depiction of transparency; on a matte white or black background, variations in transparency are indistinguishable from variations in value, rendering a value-by-alpha map conceptually identical to a bivariate choropleth map (this issue has also been considered by Roth, Woodruff, & Johnson [2010]).

The fictitious attributes presented on all maps were 'chicken consumption' and 'pizza consumption', measured along an ordinal scale (Low, Med, and High) (**Figure 3.2**). These attributes were selected to avoid depicting phenomena that would elicit a strong emotional response (such as crime rates, income, etc.), and to avoid phenomenon that participants would presume to be correlated in reality (whether

that correlation be positive or negative). These fictitious attributes were also selected to ensure that the various map types *do not* represent the information saliently (i.e., none of the map types should be more perceptually or cognitively appropriate for representing the phenomenon).

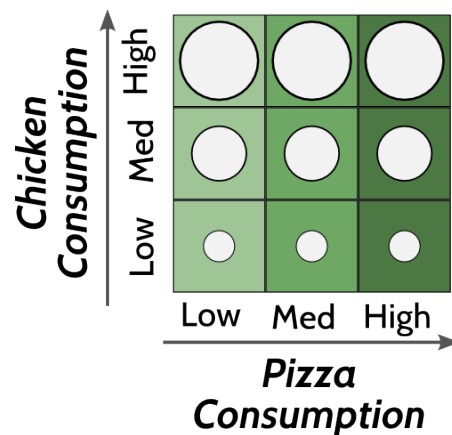


Fig 3.2 Example legend using the fictitious attributes.

Each map legend was then applied to a basemap comprised of 36 counties from western Ohio, rotated 90 degrees, and modified slightly in size and topology (**Figure 3.3**). The goal was to create a basemap that met several criterion: The enumeration units were generally consistent in size, and could be aggregated into nine compact, four-unit regions. The areal units can be allotted into nine larger, compact, four-unit *regions*. The remainder of this text will refer to the individual enumeration units as **units**, and the larger four-unit areas as **regions**. This distinction allowed for the inclusion of both Elementary (unit-level) and General

(region-level) questions in the experimental design. Subdividing the map into regions also assisted in allotting symbols onto the maps in a balanced way.

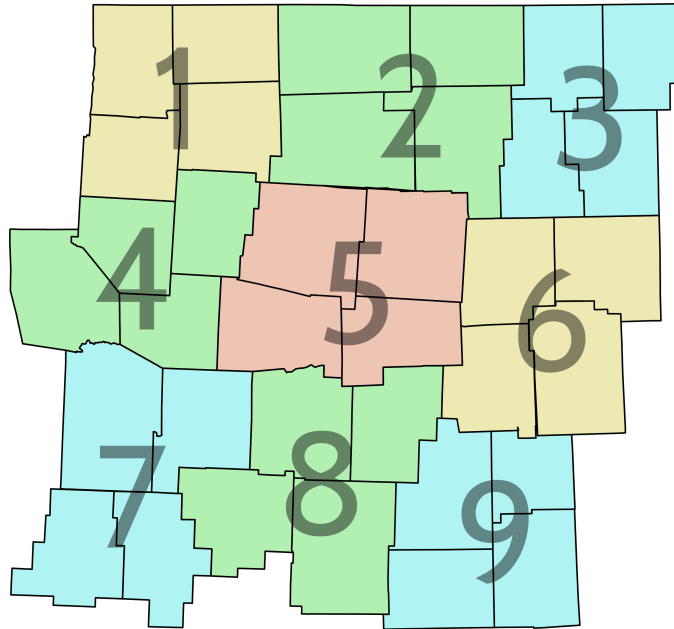


Figure 3.3. The basemap, with color distinguishing the nine regions.

From this basemap and the predetermined legend designs, the eight bivariate maps were constructed (see **Figure 3.4** for a visual representation of this process and **Figure 3.6** for a collection of the final maps). A rule for constructing regions representative of each combination of attributes was determined: two regions on the map would be High X/High Y, two would be Low X/Low Y, two Low X/High Y, two High X/Low Y, and one Med X/Med Y. A region would contain two symbols of its representative data combination, and one symbol each from the data combinations adjacent to it on the legend; for instance, a High X/High Y region would contain two

High X/High Y symbols, one Med X/High Y symbol, and one High X/Med Y symbol.

The allotment of regions on each map type was randomized and normalized such that no two maps would portray an identical distribution of symbols. The allotment of symbols to the four units within a region was randomized as well.

3.3 Procedure

The questions asked in the survey were based on the discussion of map reading tasks in **Section 2.3**. The questions considered represented a combination of three criteria:

- **Search Objective (2)**: Either *identify* (retrieve value from one unit or one region) or *compare* (assess similarity/difference between two units or two regions).
- **Search Axis (4)**: The information axis within which the user identifies or compares. As introduced in **Section 2.2.2**, these axes include *X* (the first mapped attribute), *Y* (the second mapped attribute), *Plus(+)* (positive association between attributes; High/High to Low/Low), and *Minus(-)* (negative association between attributes; High/Low to Low/High).
- **Level of Reading (2)**: tasks can be performed at either the *Elementary* level (looking at single map units) or *General* level (looking at the regions composed of multiple units).

The combination of these three factors results in 16 potential tasks (**Table 3.2**).

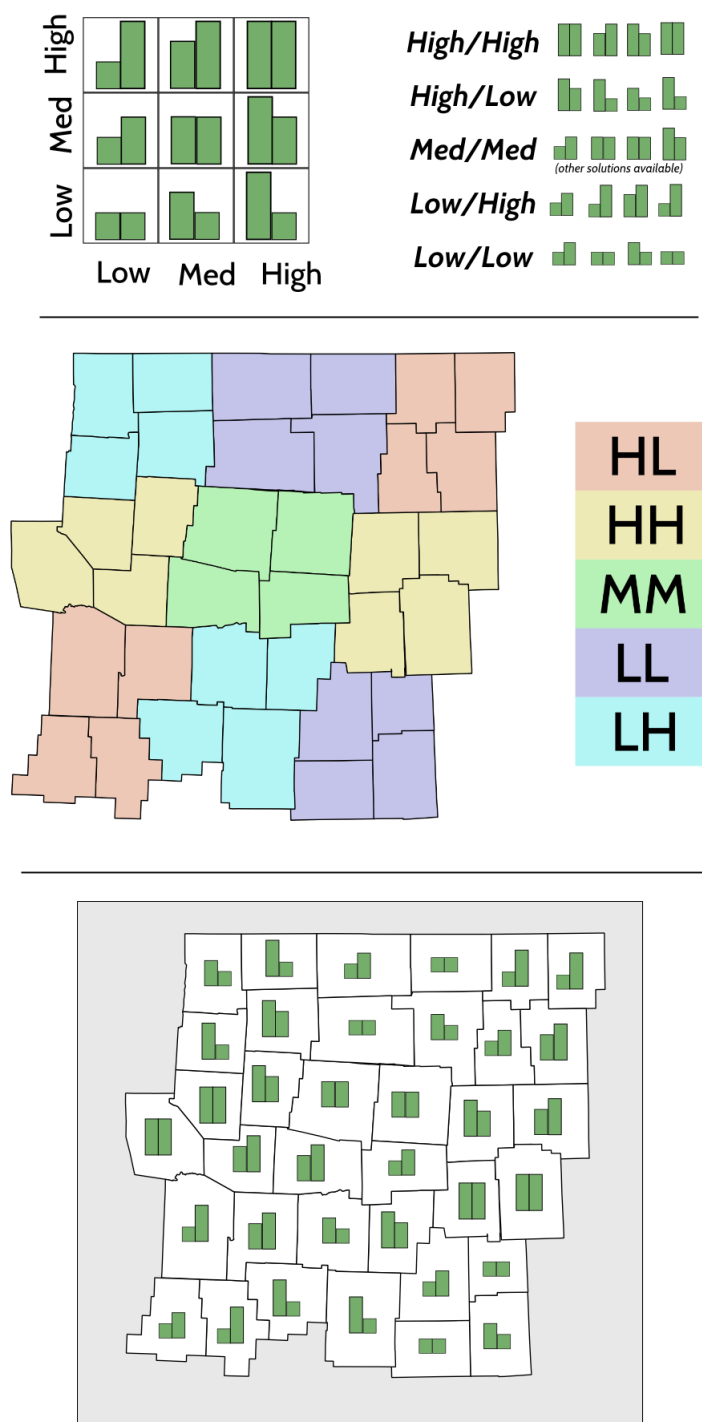


Fig 3.4. The construction of the base maps.

<i>IDENTIFY TASKS</i>				
	X	Y	+	-
Elementary	ID, Elementary X	ID, Elementary Y	ID, Elementary +	ID, Elementary -
General	ID, General X	ID, General Y	ID, General +	ID, General -

<i>COMPARISON TASKS</i>				
	X	Y	+	-
Elementary	Compare, Elementary X	Compare, Elementary Y	Compare, Elementary +	Compare, Elementary -
General	Compare, General X	Compare, General Y	Compare, General +	Compare, General -

Table 3.2 *The task taxonomy considered by the study. Only comparison tasks were ultimately included in the experiment.*

Applying all 16 tasks for each of the 8 maps would result in 128 trials total. This number of trials was deemed too large, as it would render it difficult to recruit participants, and run the risk that participants would become fatigued through the experiment and cease giving the questions their full attention. Therefore, the final survey was constrained to include only the more complex *compare* tasks, omitting the *identify* tasks. This left 8 trials for each map type, or 64 trials in total. **Figure 3.5** provides examples of each kind of task.

The survey contained 3 main portions. (1) A five-question opening survey on the participants background. (2) The main portion of the survey, consisting of a training session followed by the 64 trials, and (3) a final portion of the survey, which gauged user preference of the eight map types shown.

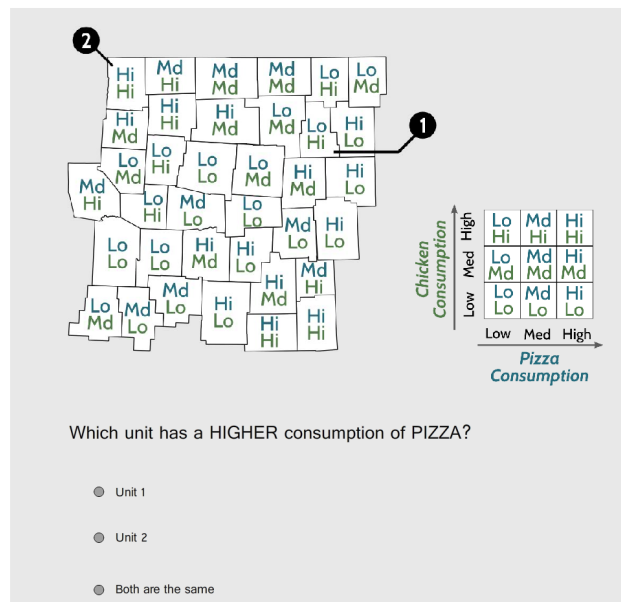
The main portion opened with a training block, designed to familiarize the participants with the user interface, map legends, and type of questions contained in the survey. The training block used a map of its own symbolization, consisting of

colored text labels that were non-analogous to any of the other map types tested (**Figure 3.5**). Responses and response times from this training block were not recorded. The different map types were presented in 8-question blocks, with their order of presentation randomized for each participant. Each block was prefaced with a familiarization screen, showing the map and the legend. These screens paused for 15 seconds before allowing the participant to continue. The order of the questions within in each block was randomized as well. For all questions, the program recorded the participants' answer as well as their response time.

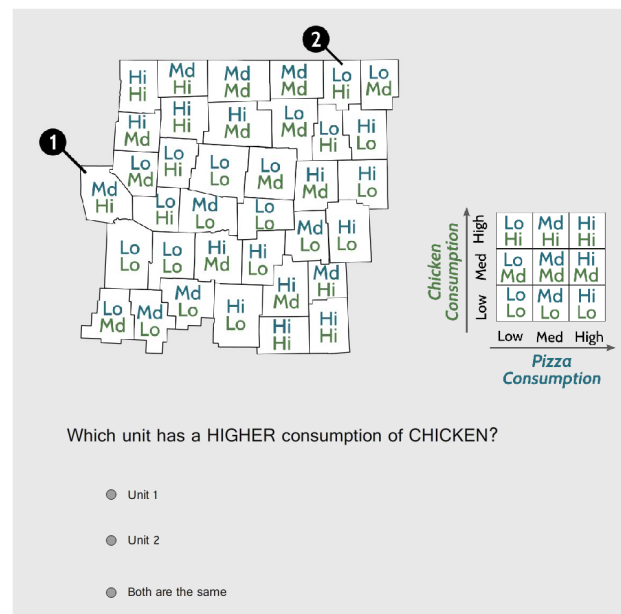
The final portion of the survey re-presented the 8 different maps used in the survey, and asked participants to rate them from 0-7 on the following scales (modified from Olson, 1981):

- *Visually Displeasing* ↔ *Visually Appealing*
- *Bad* ↔ *Good*
- *Difficult to Read* ↔ *Easy to Read*
- *Usual* ↔ *Unusual*
- *Does not show individual distributions clearly* ↔ *Does show individual distributions clearly*
- *I cannot judge the closeness of the relationship* ↔ *I can judge the closeness of the relationship*

The experiment was administered in a computer laboratory, allowing the survey to be administered to multiple participants at once in a quiet environment. The room's computers were identically imaged, and had monitors with identical size (24"), resolution (1920×1200 pixels), and contrast/brightness settings.

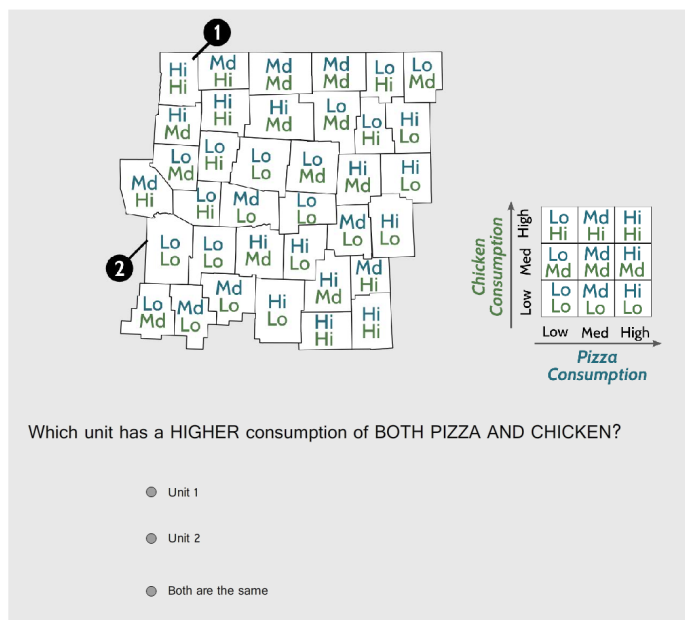


X, Elementary

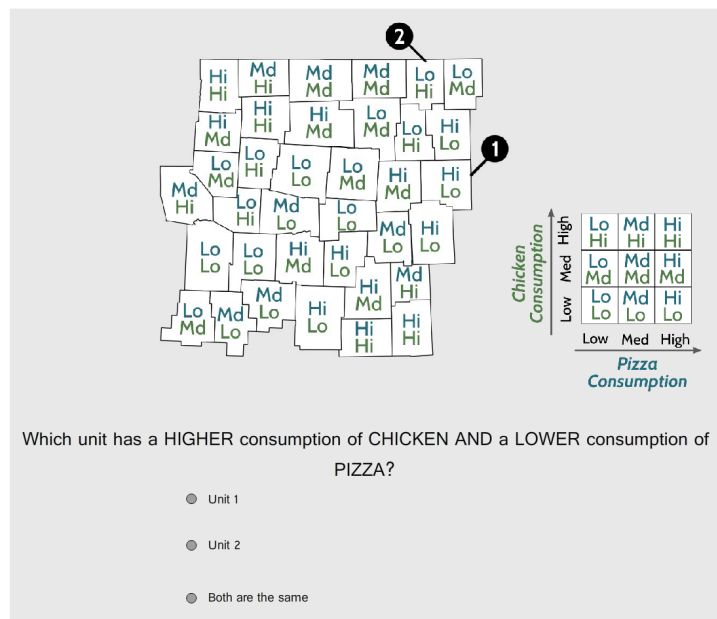


Y, Elementary

Figure 3.5a. Examples of X and Y Elementary tasks.

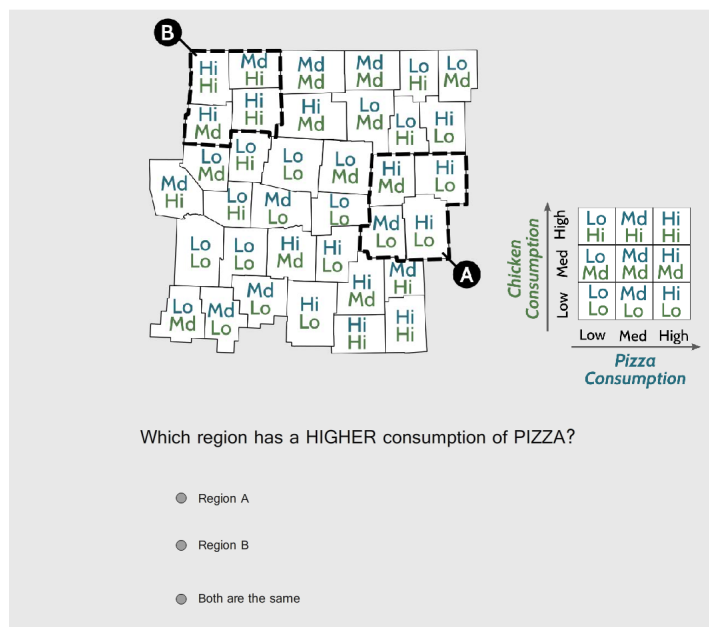


+, Elementary

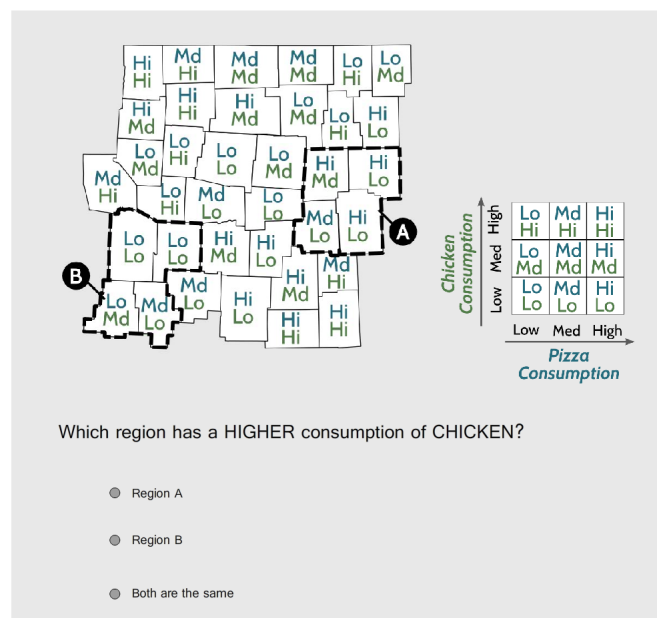


-, Elementary

Figure 3.5b. Examples of Plus(+) and Minus(-) Elementary tasks.

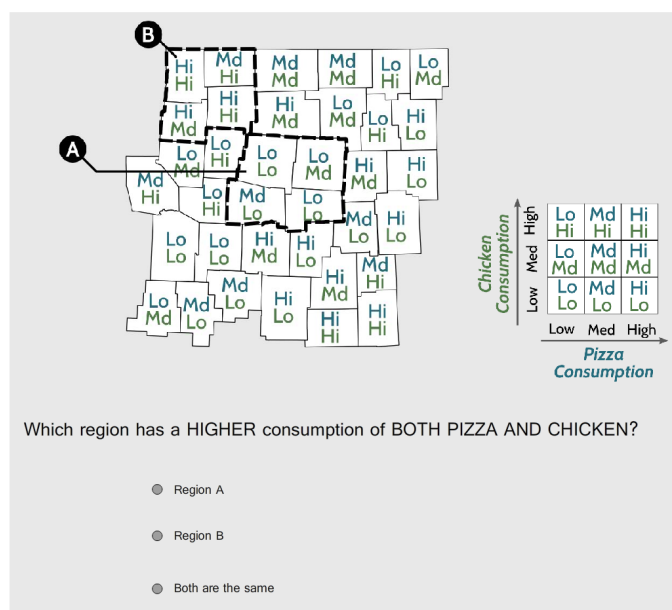


X, General

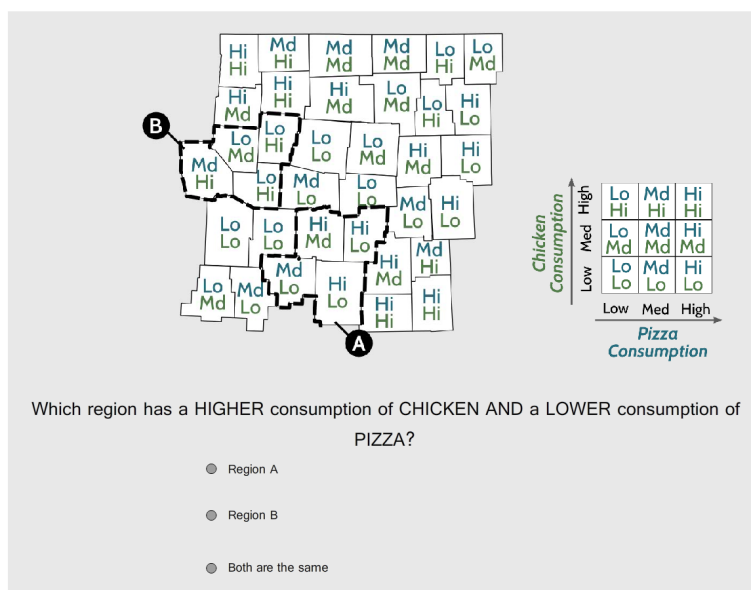


Y, General

Figure 3.5c. Examples of General X and Y tasks.



+, General



-, General

Figure 3.5d. Examples of General Plus(+) and Minus(-) tasks.



Figure 3.6a. Shaded Cartogram and Choropleth with Graduated Symbols (Separable).



Figure 3.6b. Rectangle and Bivariate Choropleth maps (Integral).

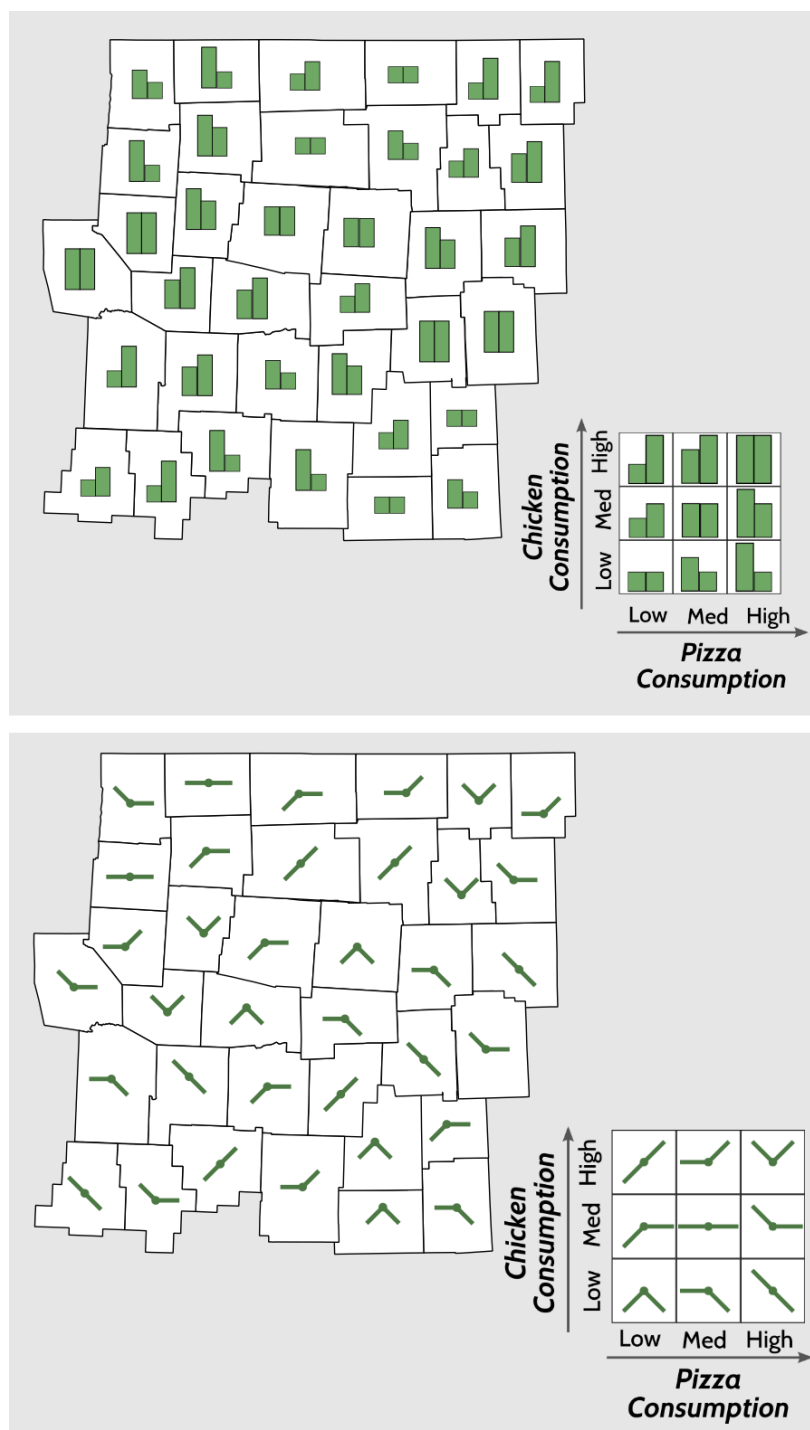


Figure 3.6c. Bar Chart and Spoke Glyph maps (Configural).

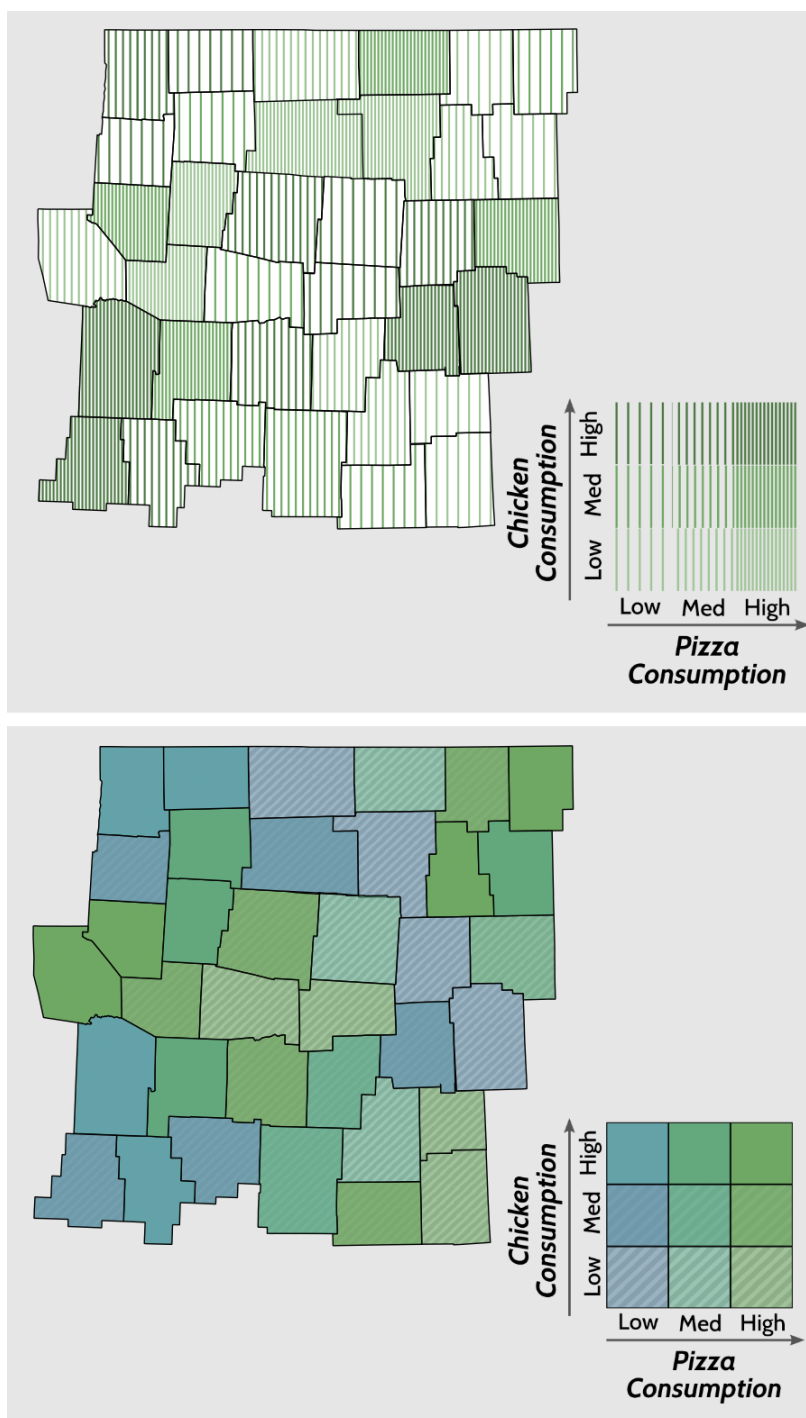


Figure 3.6d. Shaded Texture and Value By Alpha maps (Asymmetrical).

Chapter 4: Results & Interpretation

4.1 Accuracy

Participants were consistently successful in accurately answering the 64 questions. The participants' global accuracy rate was 96.1%. 17 of the 64 questions had an accuracy rate of 100% (that is, 26% of the map questions were answered correctly by every single participant). The highest and lowest accuracy rates between map types were the bivariate choropleth (99.1% accuracy) and Shaded Texture (92.7% accuracy), respectively (**Table 4.1**). The highest and lowest accuracy rates between tasks were the Elementary Plus(+) task (99.5%) and Elementary X task (93.2%), respectively. The most accurate map types by selectivity were the integral combinations (overall accuracy: 99.1%) and the least accurate were the Configural (93.9%) (**Table 4.2**). ANOVA analysis of accuracy rates between trials did not show statistically significant differences between accuracy across the different map types ($p\text{-value} = 0.064$) nor across the different conditions of selectivity ($p\text{-value} = 0.0598$), but did show statistically significant differences between the different tasks ($p\text{-value} = 0.044$).

The consistent performance in accuracy across map type indicates that all map types tested meet the minimal requirement of portraying the information, and presumably do not provide a false impression of the information. Ultimately, it is

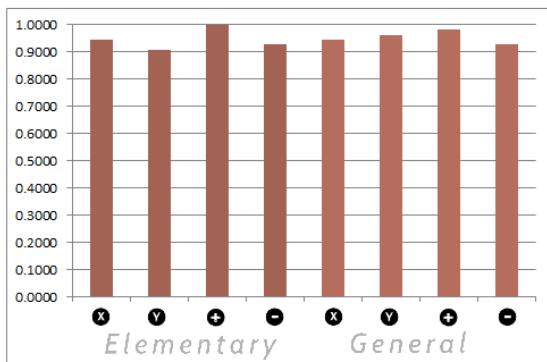
	Shaded Cartogram	Choropleth w/Grad. Symbols	Bivariate Choropleth	Rectangle Map
<i>X, Elementary</i>	0.95	0.98	0.98	0.96
<i>Y, Elementary</i>	0.91	1.00	1.00	0.98
<i>+, Elementary</i>	1.00	1.00	1.00	1.00
<i>-, Elementary</i>	0.93	0.93	0.95	0.98
<i>X, General</i>	0.95	0.95	1.00	0.98
<i>Y, General</i>	0.96	0.98	1.00	0.98
<i>+, General</i>	0.98	0.95	1.00	1.00
<i>-, General</i>	0.93	1.00	1.00	0.95
	0.95	0.97	0.99	0.98

	Value by Alpha	Shaded Texture	Spoke Glyph	Bar Chart	
<i>X, Elementary</i>	0.96	0.81	0.93	0.96	0.942
<i>Y, Elementary</i>	0.95	0.89	0.96	0.96	0.957
<i>+, Elementary</i>	0.98	0.98	1.00	1.00	0.995
<i>-, Elementary</i>	0.98	0.96	1.00	0.95	0.959
<i>X, General</i>	0.93	0.98	0.85	0.91	0.943
<i>Y, General</i>	0.98	0.96	0.96	0.98	0.977
<i>+, General</i>	0.98	1.00	0.95	1.00	0.982
<i>-, General</i>	0.96	0.91	0.85	0.96	0.945
	0.97	0.94	0.94	0.97	Averages

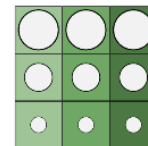
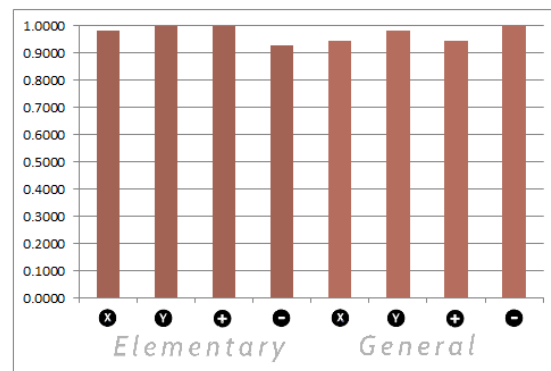
Table 4.1. Accuracy by map type and task (divided for legibility at print size).

	Separable	Integral	Asymmetrical	Configural
<i>X, Elementary</i>	0.945	0.982	0.964	0.927
<i>Y, Elementary</i>	0.909	1.000	0.945	0.964
<i>+, Elementary</i>	1.000	1.000	0.982	1.000
<i>-, Elementary</i>	0.927	0.945	0.982	1.000
<i>X, General</i>	0.945	1.000	0.927	0.855
<i>Y, General</i>	0.964	1.000	0.982	0.964
<i>+, General</i>	0.982	1.000	0.982	0.945
<i>-, General</i>	0.927	1.000	0.964	0.855
Average	0.950	0.991	0.966	0.939

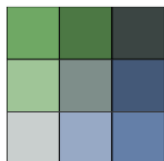
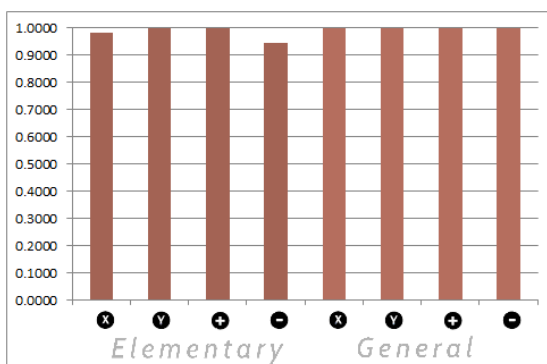
Table 4.2. Table of accuracy rates by selectivity and task.



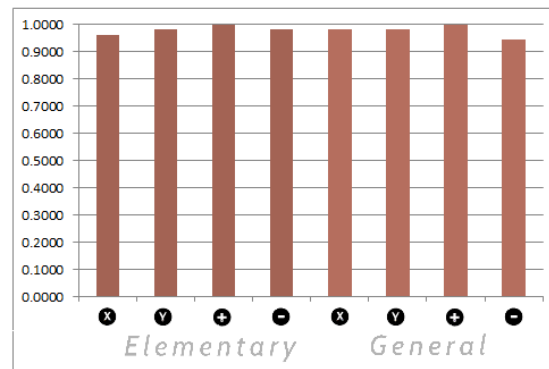
Shaded Cartogram : Separable



Choropleth w/ Grad. Symbols: Separable

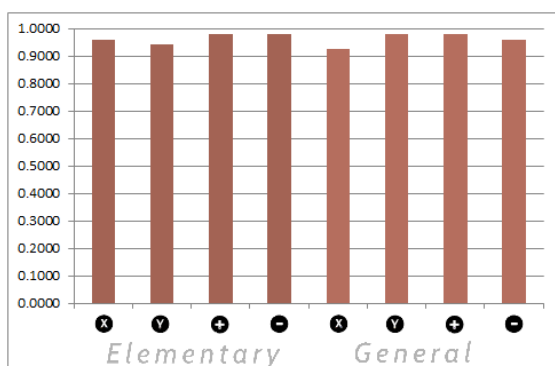


Bivariate Choropleth: Integral

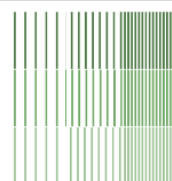
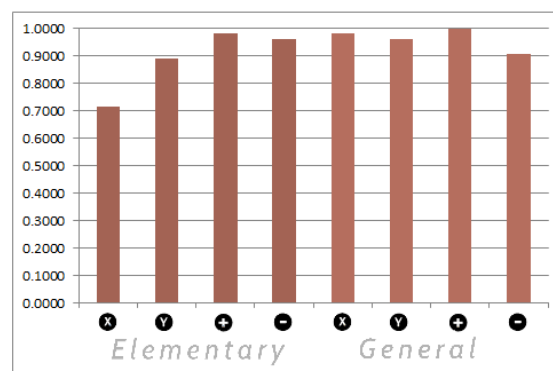


Rectangle Map: Integral

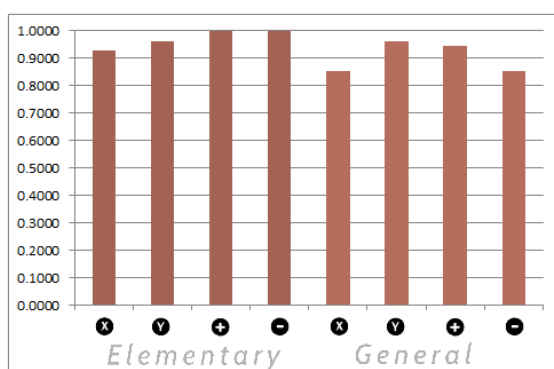
Figure 4.1a. Accuracy rates for separable and integral combinations.



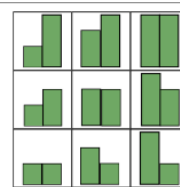
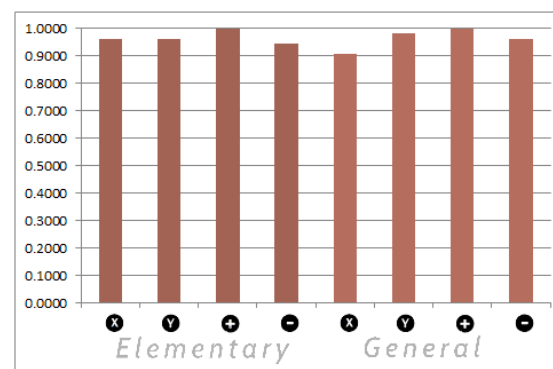
Value By Alpha: Asymmetrical



Shaded Texture: Asymmetrical



Spoke Glyph: Configural



Bar Chart: Configural

Figure 4.1b. Accuracy rates for asymmetrical and configural map types.

difficult to form any prescriptive claims about the map types based upon the accuracy results. Variations in the various participants' accuracy rates between the different map users carried as much weight as the variations in accuracy between the different map types. The global accuracy rate (that is, combining every participants' response to every question) was 96.1%. The standard deviation of all participants' accuracy rates was 0.0395. The standard deviation of accuracy rates across the 64 map trials was 0.0452.

It is challenging to explain the difference in accuracy performance across the eight different tasks. The best-performing tasks, in terms of average performance across map type, were questions along the Plus(+) axis (the Elementary and General Plus(+) axis questions had an overall accuracy rate of 99.5% and 98.2%, respectively). Due to the phrasing of the Plus(+) axis questions, and the nature of identifying positive association, the Plus(+) axis questions avoid what could be a common bivariate map reading error: forgetting which visual variable encodes which statistical variable. This mistake would cause a wrong answer when answering a question in the X, Y, or Minus(-) axes, but not the Plus(+) one.

An alternative explanation for the accuracy differences across task is the influence of an outlier. The Elementary X task had accuracy rates above 90% for most map types, except for the shaded texture map, which had an accuracy rate of 72% for that task (the lowest accuracy rate of any combination of map type/task). The relative poor performance on this question may simply be an artifact of the

specific way this symbol was constructed in this study. At the thinnest lines of the shaded texture, it is challenging to distinguish the variation in the lines' darkness; a stronger contrast between the lightest and darkest values could have alleviated any misreadings.

4.2 Response Time

Response times (RTs) varied noticeably across map type, task, and selectivity. Across the different map types, the lowest median RT was found in the rectangle map (20.7 seconds) and the highest was found in the spoke glyph (36.1 seconds).

Response times (RTs) showed statistically significant differences across both map type, map selectivity, and task ($P < 2e^{-16}$, using ANOVA). These differences indicate that the choice of map type has important impacts on how intuitively users are able to extract various forms of information from the map.

Between the different map types, a pairwise comparison of means (Tukey HSD test) found statistically significant differences (at $p < 0.05$) in 18 out of the 28 possible comparisons between map types (**Table 4.5**). These differences hold true when aggregating each map's results to compare between selectivities, with every condition of selectivity being statistically significant when compared against each other (again, using Tukey HSD at $p < 0.05$) (**Table 4.6**).

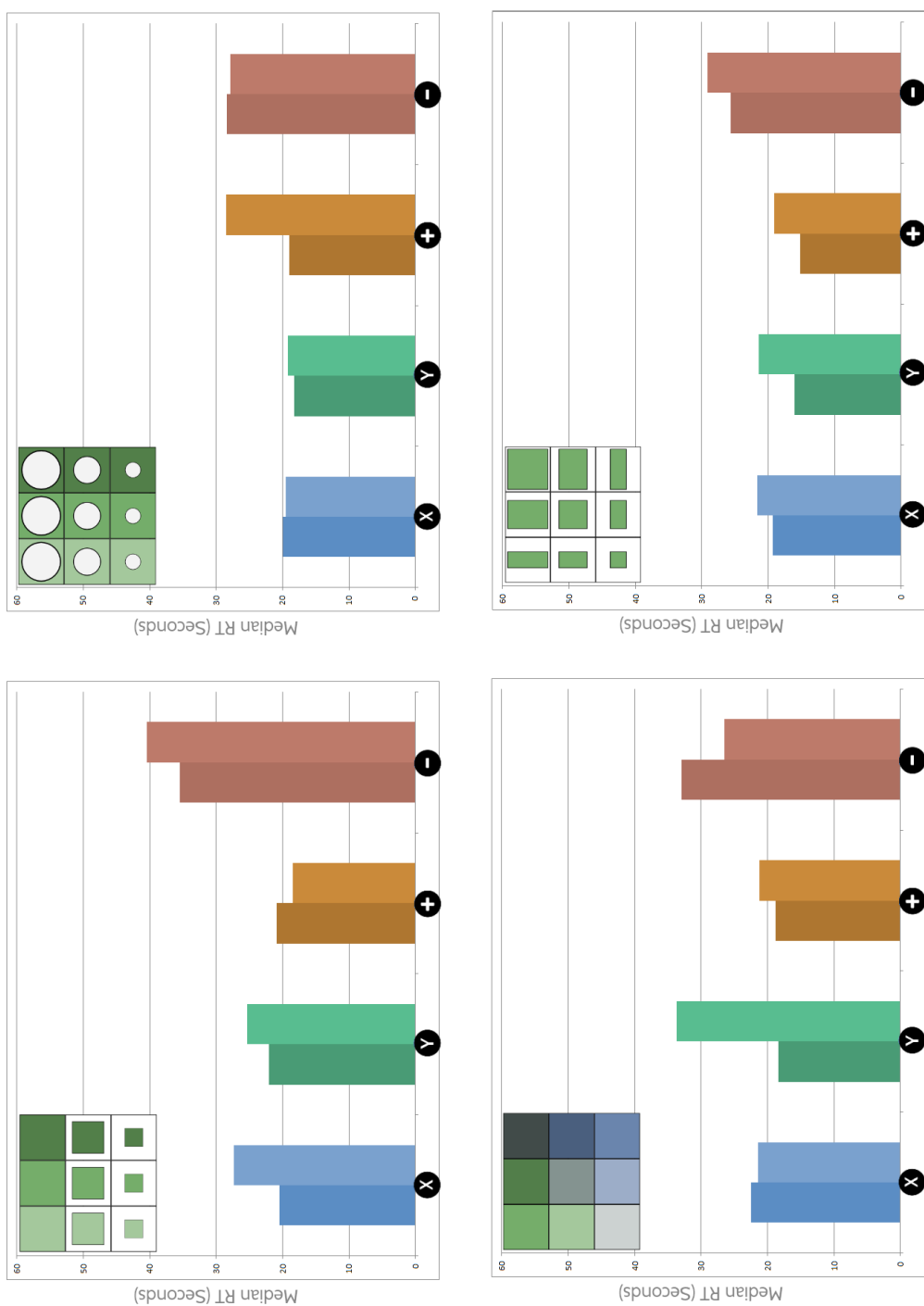


Figure 4.2a. Median RTs (secs) for separable and integral map types, by task. Left side of bar represents Elementary level, right side Intermediate level.

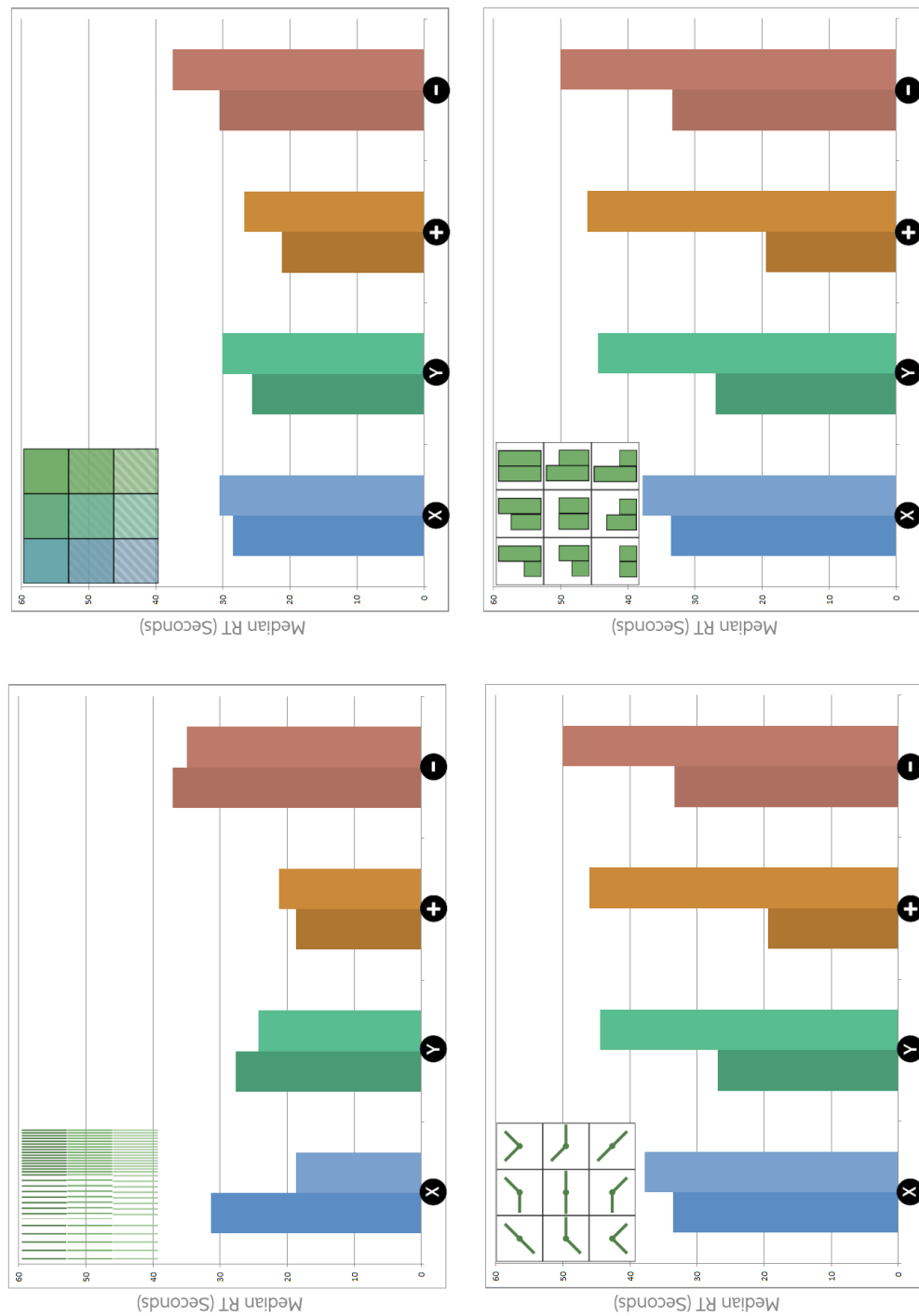


Figure 4.2b. Median RTs (secs) for asymmetrical and configurational map types, by task. Left side of bar represents Elementary level, right side Intermediate level.

	Shaded Cartogram	Choropleth w/Grad. Symbols	Bivariate Choropleth	Rectangle Map
<i>X, Elementary</i>	20.85	20.06	22.10	19.31
<i>Y, Elementary</i>	23.46	19.38	20.15	16.52
<i>+, Elementary</i>	21.25	19.38	19.69	15.40
<i>-, Elementary</i>	35.83	28.38	33.02	25.67
<i>X, General</i>	27.81	19.58	21.19	21.73
<i>Y, General</i>	25.81	19.08	33.79	21.67
<i>+, General</i>	18.46	28.96	21.00	19.31
<i>-, General</i>	40.52	28.98	27.35	29.58
	26.75	22.97	24.79	21.15

	Value by Alpha	Shaded Texture	Spoke Glyph	Bar Chart	
<i>X, Elementary</i>	28.73	33.40	36.44	23.56	20.58
<i>Y, Elementary</i>	25.46	28.27	28.38	24.94	19.88
<i>+, Elementary</i>	21.60	18.42	19.69	13.48	18.93
<i>-, Elementary</i>	31.56	36.79	34.75	32.29	30.72
<i>X, General</i>	31.31	19.02	38.35	28.63	22.58
<i>Y, General</i>	30.56	25.15	47.96	26.15	25.09
<i>+, General</i>	26.83	21.44	47.42	18.04	21.93
<i>-, General</i>	39.08	35.71	53.65	57.56	31.61
	29.39	27.27	38.33	28.08	Average

Table 4.3. Median response times (in seconds) across the 64 trials.(table divided for legibility at print size)

	Separable	Integral	Asymmetrical	Configural
<i>X, Elementary</i>	24.55	24.88	35.02	32.52
<i>Y, Elementary</i>	25.29	23.04	29.55	29.88
<i>+, Elementary</i>	24.12	18.78	22.96	20.45
<i>-, Elementary</i>	39.40	32.27	41.19	35.12
<i>X, General</i>	29.83	25.01	33.17	39.85
<i>Y, General</i>	27.56	29.93	32.83	43.93
<i>+, General</i>	28.62	22.37	29.25	36.14
<i>-, General</i>	38.51	32.82	46.17	67.24
Average	29.73	26.14	33.77	38.14

Table 4.4. Median response time (in seconds) across the map types and trials, aggregated to selectivity.

<i>Choropleth w/Grad.</i>	< 0.001		
<i>Bivariate Choropleth</i>	0.00491	0.97051	
<i>Rectangle Map</i>	< 0.001	0.96503	0.42442
<i>Value by Alpha</i>	0.88367	< 0.001	< 0.001
<i>Shaded Texture</i>	0.99634	0.00154	0.05442
<i>Spoke Glyph</i>	< 0.001	< 0.001	< 0.001
<i>Bar Chart</i>	1	< 0.001	0.00293
	<i>Shaded Cartogram</i>	<i>Choropleth w/Grad. Symbols</i>	<i>Bivariate Choropleth</i>

<i>Value by Alpha</i>	< 0.001			
<i>Shaded Texture</i>	< 0.001	0.43969		
<i>Spoke Glyph</i>	< 0.001	< 0.001	< 0.001	
<i>Bar Chart</i>	< 0.001	0.92918	0.98993	< 0.001
	<i>Rectangle Map</i>	<i>Value by Alpha</i>	<i>Shaded Texture</i>	<i>Spoke Glyph</i>

Table 4.5. *p*-values of pairwise comparison of mean response time. (Table divided for legibility at print size). Statistically significant results (at $p < 0.05$) are highlighted in green.

<i>Integral</i>	0.00464		
<i>Configural</i>	< 0.001	< 0.001	
<i>Asymmetrical</i>	< 0.001	< 0.001	< 0.001
	<i>Separable</i>	<i>Integral</i>	<i>Configural</i>

Table 4.6. *p*-values of pairwise comparison of mean response time, organized by selectivity. Statistically significant results (at $p < 0.05$) are highlighted in green.

	Average Median RT (across all map types)	Median RT \pm 0.25	Map types with median RTs < Global Median RT \pm 0.25	Map types with median RTs > Global Median RT \pm 0.25
X_i Elementary	25.29	18.97 – 31.61	<i>none</i>	Shaded Texture (median 31.9, $p = 4.81e-05$), Spoke Glyph (median 34.66, $p = 0.000124$)
Y_i Elementary	23.31	17.47 – 29.11	Rectangle Map (median 17, $p = 0.0479$)	<i>none</i>
$+$ Elementary	18.66	14 – 23.33	Bar Chart (median 13.58, $p = 0.0137$)	<i>none</i>
$-$ Elementary	32.04	24.03 – 40.05	<i>none</i>	<i>none</i>
X_i General	25.93	19.44 – 32.4	Shaded Texture (median 18.7, $p = 0.00016$)	Spoke Glyph (median 38.29, $p = 1.46e-06$)
Y_i General	28.76	21.56 – 35.94	Choropleth w/ Grad. Symbols (median 19.21, $p = 0.00016$)	Spoke Glyph (median 47.66, $p = 2.19e-15$)
$+$ General	25.14	18.84 – 31.375	Shaded Cartogram (median 18.46, $p = 0.0868$), Bar Chart (median 18.17, $p = 0.00239$)	Spoke Glyph (median 41.13, $p = 2e-16$)
$-$ General	38.72	29.03 – 48.4	Choro. w/ Grad. (median 27.92, $p = 0.000444$), Bivariate Choropleth (median 26.5, $p = 0.00304$)	Spoke Glyph (median 52.58, 0.00304), Bar Chart (median 59.08, $p = 7.07e-08$)

Table 4.7. Map types that differed from the global median within a given task.

These measures provide information only on the global differences in reaction time across the different map types: examining the variation of the maps' performance across the eight different tasks shows readily apparent nuances (**Figure 4.2**). To elucidate meaningful effect sizes in reaction times between the map types and tasks, additional statistical tests were run if any map type deviated 25% or more from all maps' median RT for that task (**Table 4.7**). To illustrate with a specific example: averaging the median RTs for the general-level Y task, across all maps, results in an average median RT of 28.76 seconds. $28.76 \pm .25$ creates a range of 21.56 seconds to 35.94 seconds. Any map type that had a median RT outside these bounds in the General Y task had its reaction times compared with the aggregated reaction times of all other maps within that task.

In analyzing **Table 4.7**, The performance of the spoke glyph deserves attention: it performed considerably slower than the average map in five out of the eight tasks, and in all cases these results were statistically significant at $p < 0.05$. This includes performing worse than the other maps in all four general-level questions. In all, participants responded to the spoke glyph 8.76 seconds slower than average in the Elementary X task, 12.4 seconds slower in the General X task, 18.9 seconds slower in the General Y task, 16 seconds slower in the General Plus(+) task, and 13.9 seconds slower in the General Minus(-) task. There are two likely explanations for the spoke glyph's markedly poor performance. First, orientation itself may be unintuitive for portraying this set of ordinal data. One way to

conceptualize the spoke glyph symbol is to read it similarly to the speedometer of a car, with a bar pointing upward to indicate a high attribute value. Conjecturally, viewers may instead associate the symbol's shape with the hands of an analog clock, an association that would not assist (and may in fact actively interfere) with the ability to interpret the orientation of the spoke glyph's bars with the concept of High/Med/Low they were intended to represent. Alternatively, it may be that the spoke glyph simply fails to provide any compelling gestalt dimensions for the reader to attend. When assembled together on the map (**Figure 4.3**), the spoke glyphs fail to form any immediately intuitive emergent dimensions, but rather a chaotic assemblage of lines, angles, and whitespace. This explanation would especially account for the spoke glyph's poor performance in general map reading questions. Whatever the reason, the fact that reaction times to the spoke glyph were so strongly and universally slow calls into question the continued use of this symbol as a bivariate mapping solution.

In the Elementary X task, the shaded texture and spoke glyph maps were slower than the rest of the map types tested. The shaded texture map had RTs 6.6 seconds higher than average in this task. Contrast difficulties inherent in the shaded texture's symbol design (as mentioned in **Section 4.1**) are a probable contributor. Curiously, the shaded texture map performed worse than the other maps in the Elementary X task, but performed better than the other maps in the General X task by 7.2 seconds on average. This difference is potentially due to viewers being able

to visually aggregate the closeness of the lines, forming what looks akin to a more traditional and intuitive visual cue: value steps (**Figure 4.4**).



Figure 4.3. The spoke glyph map.

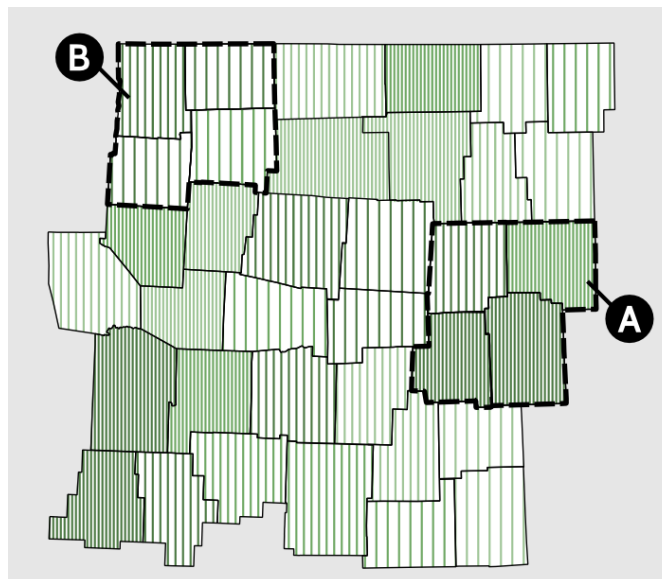


Figure 4.4. The General X shaded texture map.

The quick reaction times to the rectangle map in the Elementary Y task (6.3 seconds faster than average) are surprising, especially given that this effect was not seen in the rectangle map's Elementary X task. Assumedly, the height of a rectangle would be an equally effective means of encoding information as its width. This performance in Elementary Y was just barely statistically significant at the 0.05 confidence level ($p = 0.0479$), so this difference in performance may simply be the product of sampling error. Alternatively, participants who completed the bar chart block before the rectangle map may have been primed to seek out variations in symbol height. In the General Y task, the spoke glyph performed poorly (as previously discussed), and the choropleth with graduated symbols performed better than the others as a whole (median RT 9.5 seconds lower than average). This may be reflective of the choropleth with graduated symbol's separability: viewers were able to attend to the graduated symbols with little interference from the underlying choropleth.

Two map types had particularly low reaction times in the General Plus(+) task: the shaded cartogram (median RT 6.7 seconds less than average) and the bar chart (median RT 7 seconds less than average). Of these two, only the bar chart's differences were statistically significant (at $p < 0.05$). Visual cues in the Plus(+) axis were hypothesized to be strongest in integral combinations, so it's surprising that the bar chart (a configural combination) provided the best reaction times in the Plus(+) axis tasks, which it did at both the Elementary and General level of reading. Despite

not being integral, the bar chart still contains a strong emergent dimension along the Plus(+) axis: the area of the symbol, in a similar way to the integral rectangle map.

Finally, the General Minus(-) task was well supported by both the choropleth with graduated symbols (median RT 10.8 seconds less than average) and the bivariate choropleth (12.2 seconds less), and poorly supported by the spoke glyph (13.9 seconds higher than average) and bar chart (20.4 seconds higher). The choropleth with graduated symbols and the bivariate choropleth supported this task in different ways. In the case of the choropleth with graduated symbols, the separability of the two variables provides an uninhibited means to locate each attributes' value on the legend, and answer the question by attending to the attributes individually. In the case of the bivariate choropleth, it contains a very strong visual cue along the Minus(-) axis: the greenness/blueness of the symbol. Viewers needed only to match the color chips with their representative version in the legend, avoiding the need to attend to the data variables individually.

Map Type	Axis	Median RT (Elementary)	Median RT (General)	p value
<i>Choropleth w/ Graduated Symbols</i>	+	19.00	28.96	7.22E-005
<i>Bivariate Choropleth</i>	Y	19.83	33.88	6.56E-005
<i>Shaded Texture</i>	X	31.92	18.71	1.70E-005
<i>Spoke Glyph</i>	Y	27.83	47.67	1.13E-005
<i>Spoke Glyph</i>	+	19.79	47.13	1.93E-009
<i>Spoke Glyph</i>	-	34.46	53.58	0.000149
<i>Bar Chart</i>	-	31.50	59.08	1.35E-007

Table 4.8. *Combinations of map type/task wherein the General level differed from the Elementary level at $\pm 25\%$.*

Examining variations in performance between Elementary-level and General-

level tasks within a map type, a similar technique was used as the examination presented in **Table 4.7**. Map type/tasks whose median reaction times varied by more than 25% between the Elementary and General levels are assembled in **Table 4.8**. ANOVA tests were then run treating Elementary-level responses and general-level responses for each combination of map type/search axis as independent populations. All of the tested differences were statistically significant at $p < 0.05$.

In six out of the seven tasks tested, reaction times were slower moving from the Elementary task to the General one. This may represent a technique on the participants' behalf to cautiously answer the general-level questions by attending individually to the four enumeration units within each region, rather than attempt to use gestalt dimensions to create an 'at-a-glance' understanding of the attribute values within the region.

Some of the most extreme disparities in reaction times between the Elementary/General level occurred in the configural map types within the Minus(-) axis. In both the spoke glyph and the bar chart, participants took more than 50% longer to respond to the General Minus(-) task compared to the Elementary Minus(-) task. A possible explanation can be found in how the configural combinations visually encode the Minus(-) axis: by a 'leftedness' vs. 'rightedness' of the symbol. At the Elementary level, the viewer need only to determine which side of the symbol is higher, and relate that to which side of the symbol encodes which attribute. At the General level, the viewer must attempt to aggregate the leftedness/rightedness of

several of these symbols, and the fact that these symbols are located to the left and right of each other on the map necessarily provides interference (**Figure 4.5**).

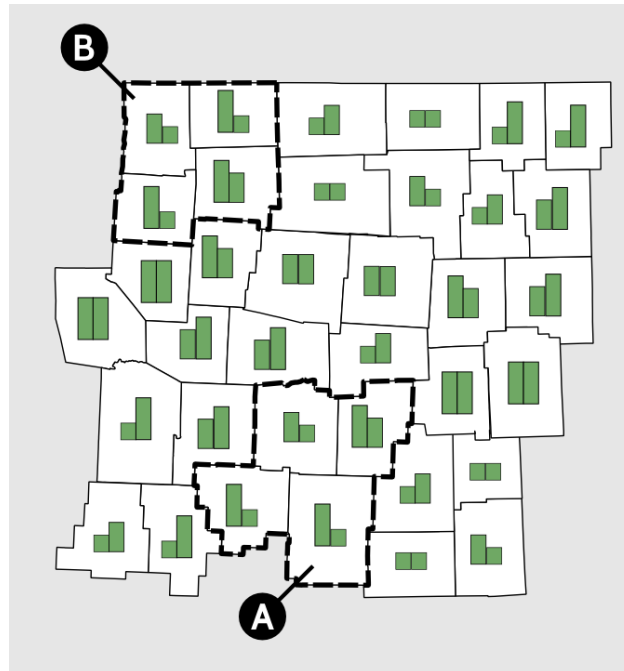


Figure 4.5. *The General Minus(-) task for the bar chart map.*

4.3 Influence of User Expertise

Participants' accuracy and response times varied little according to user expertise. The difference in average reaction times between those who have and those who haven't taken coursework in Cartography/GIS is roughly three seconds (**Table 4.9**). Participants reporting an educational background had an overall accuracy roughly half of a percent (0.04%) better than those without. Using ANOVA to test these differences at $p = 0.05$ reveals no statistically significant results. Nearly

identical results occur using the other major measure of expertise, work experience. Again, no statistically significant differences were found between those with and those without work experience in Cartography/GIS.

Group	Average RT	Average Accuracy
<i>Have taken coursework</i>	33.1	0.955
<i>Have not taken coursework</i>	29.9	0.951
P-value	0.237	0.698

Group	Average RT	Average Accuracy
<i>Has had work experience</i>	33.2	0.951
<i>Has not had work experience</i>	30.5	0.956
P-value	0.296	0.613

Table 4.9. Accuracy and reaction time differences according to two measures of user expertise.

Group	n	Mean response time	Mean accuracy rate
<i>Has both education and job experience, rates self as familiar with maps (>4)</i>	25	33.36	0.952
<i>Has neither education nor job experience, rates self as unfamiliar with maps (<5)</i>	13	30.92	0.951

Table 4.10. Comparison of RT/accuracy for the extrema in expertise.

Even at the most extreme difference in user expertise, little difference between participants is seen. **Table 4.10** shows only the most experienced and least experienced participants in the study: those who reported both educational and work experience in Cartography/GIS, and self-rated themselves as familiar with maps (5 or higher on the 7-point Likert scale), and those who reported neither educational nor work experience in Cartography/GIS, and self-rated themselves as unfamiliar with maps (4 or lower on the 7-point Likert scale). Once again, the effect size

between these two groups is minimal: mean reaction times varied by less than 2.5 seconds, and both groups were within a single percentage point of each other in accuracy rates.

There are several ways to interpret these results. Although those involved in the fields of Cartography/GIS are more likely to encounter bivariate maps like the ones used in the study, it is uncommon for any coursework or job training to actually instruct individuals on techniques to extract information from a bivariate thematic map. In other words, there exists little in the way of fostering *map literacy* specific to bivariate maps. As examined throughout **Section 2.2.2**, bivariate maps contain graphical cues that can assist in efficiently extracting information from them, but these visual cues exist subtextually in the map. A map reader, even one with expertise in map use, may not notice the existence of these 'visual shortcuts', and therefore be unable to exploit them to extract information from the map more efficiently.

Alternatively, the participants in the study may not have differed in expertise enough to produce meaningful results. It must be noted that, despite ostensibly collecting a broad range of participants, almost every participant in this study was recruited from within an institution of higher learning, and within a department dedicated to Geography. Even the least expert participants represented in this study can be presumed to be more privileged in their education, spatial reasoning, and ability to use technology compared to much of the human population.

4.4 User Preference

	Shaded Cartogram		Choro. w/ Grad. Symb		Bivariate Choropleth		Rectangle Map	
	Mean	St.Dev	Mean	St.Dev	Mean	St.Dev	Mean	St.Dev
<i>Visually Displeasing <-> Visually Appealing</i>	4.0	1.6	4.8	1.5	5.7	1.4	3.7	1.6
<i>Usual <-> Unusual</i>	4.5	1.5	3.6	1.7	2.5	1.7	4.6	1.6
<i>Difficult to Read <-> Easy to Read</i>	4.4	1.6	5.4	1.6	5.0	1.7	4.1	1.8
<i>Does not Show Individual Distribution Clearly <-> Does " "</i>	4.5	1.6	5.6	1.5	4.8	1.7	4.4	1.8
<i>I Cannot Judge the Closeness of the Relationship <-> I Can " "</i>	4.8	1.5	5.5	1.5	5.2	1.5	4.7	1.7
<i>Bad overall <-> Good Overall</i>	4.5	1.5	5.3	1.5	5.1	1.3	4.2	1.6

	Value by Alpha		Shaded Texture		Spoke Glyph		Bar Chart	
	Mean	St.Dev	Mean	St.Dev	Mean	St.Dev	Mean	St.Dev
<i>Visually Displeasing <-> Visually Appealing</i>	5.3	1.6	2.4	1.6	2.4	1.4	3.7	1.4
<i>Usual <-> Unusual</i>	4.5	1.7	5.0	1.6	5.6	1.6	3.9	1.5
<i>Difficult to Read <-> Easy to Read</i>	3.9	1.6	2.5	1.4	2.5	1.7	4.3	1.9
<i>Does not Show Individual Distribution Clearly <-> Does " "</i>	3.8	1.7	2.9	1.7	3.4	2.1	4.9	1.7
<i>I Cannot Judge the Closeness of the Relationship <-> I Can " "</i>	3.9	1.5	3.2	1.7	3.7	1.9	5.0	1.6
<i>Bad overall <-> Good Overall</i>	4.0	1.4	2.6	1.5	2.7	1.6	4.3	1.6

Table 4.11. Likert scale results: mean and standard deviation for participants' scoring of the map types along various scales (all running 1-7).

The closing questions on user preference showed clear differences among the eight map types. Kruskal-Wallis (a non-parametric version of ANOVA) found statistically significant differences between how users ranked the map types (p value = $2.2e^{-16}$). **Table 4.11** provides a summary of the Likert scores.

Using mean scores, the bivariate choropleth was judged to be the most visually appealing (mean score 5.7, with 7 being most appealing) and most usual (mean score 2.5, with 1 being most usual). The choropleth with graduated symbols

was judged to be the easiest to read (mean score 5.4, with 7 being easiest to read), a sentiment that also held true when asked about ease of reading individual distributions of the attributes (mean score 5.6) and determining the closeness of their relationship (mean score 5.5). The most favorably received maps, based on the 'bad overall/good overall' ranking, were the choropleth with graduated symbols (mean score 5.3) and the bivariate choropleth (mean score 5.1). Using the same scale, the least favorably received maps were the shaded texture (mean score 2.6) and spoke glyph (mean score 2.7).

There is little variation in how participants responded to the questions within each map type: that is, if a participant liked or disliked a map type, they would rate it similarly high or low regardless of the question asked. This fact is of particular interest with regards to the "*shows individual distributions clearly*" and "*I can judge the closeness of the relationship*" scales. Theoretically, based on knowledge of selective attention, the integral combinations would perform better at showing relationships than individual distributions, and vice-versa for separable combinations. In these scores, the participants reported little difference in how successfully these map types represent the individual attributes versus the relationship between those attributes. This may be further evidence that participants were unable to notice or capitalize on the emergent visual dimensions that exist within the maps, as mentioned in **Section 4.3** above.

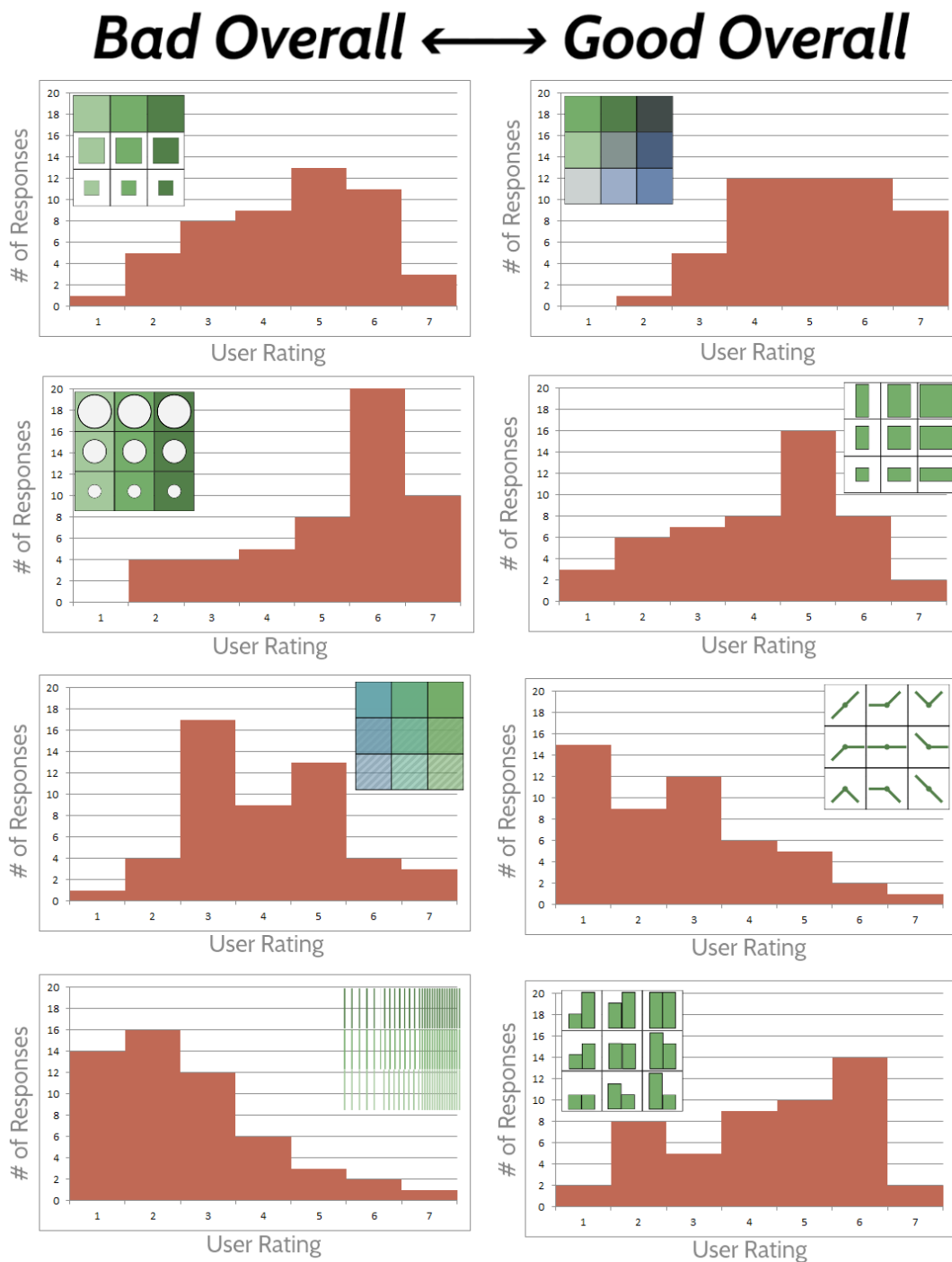


Figure 4.6. Histograms of user responses to the 'bad overall/good overall' Likert scale. Well-received map types would have most responses to the right end of the scale.

Chapter 5: Conclusions

5.1 Summary

Despite being common in cartography, bivariate thematic maps are poorly understood; few academic articles examine them empirically, cartography textbooks and curricula do not discuss them systematically, and in general there is little understanding of how to design bivariate symbol sets that effectively portray a given set of information.

This research sought to assist in the resolution of these issues in two ways. First, to better understand how bivariate maps 'work' by exploring the solution space for bivariate maps, understanding their unique perceptual characteristics (via the theory of selective attention), and examining the nature of how these various solutions encode relational information about the mapped phenomena. Secondly, this research applied those insights to a controlled study designed to identify various potential influences on bivariate map reading: the type of bivariate map used (including the selective attentive characteristics of that map type), the tasks in which the reader performs using the map, and the expertise of the map reader.

5.2 Experimental Findings

The controlled experiment performed in this study is an attempt to adapt the methodology of Selective Attention research (namely, speeded classification) with methodologies common in Thematic Cartography research (performance testing).

Results indicate that while participants were ultimately able to successfully extract information from all of the eight different map types, there were meaningful differences in how well the tested map types supported various forms of information retrieval.

Despite hesitations about the utility of bivariate maps (described in **Section 1.2**), the participants in this survey were consistently successful in accurately answering the questions presented to them (overall accuracy rate was 96.1%). There were also several map types that users rated largely positively in their capacity to read and understand the information on the map (based on the Likert scores in **Section 4.4**). Essentially, although some map types were more intuitive to read than others (according to reaction time, with the spoke glyph proving the least intuitive), no map was too visually complex to be understood. It should be recognized that the structured, task-specific way participants interacted with the maps in this study may have influenced their impressions of the maps' utility. Task-specific map reading engages different visual and cognitive activities than the unstructured, spontaneous kinds of map reading that is more likely to occur when people encounter maps in everyday life (Antes & Chang 1990). Nevertheless, the results of this study should dispel some hesitations about the utility of bivariate maps: when thoughtfully designed, bivariate maps are capable of being largely successful in their communication goals.

Selective Attention theory provided some insights into the performance of the

eight map types, but could not account for all results seen. There were statistically significant differences between selectivities in terms of their reaction times, but not in terms of accuracy rates. A given map type did not always perform identically to the other map type with the same selectivity; the disparities in performance between the configural solutions (bar chart and spoke glyph) provide a good example. Because of these intra-selectivity differences, it is challenging to draw broad conclusions about the four conditions of selectivity based on the findings of this study. Results that we would expect to see due to knowledge of perception did not always materialize; the best performance in the General Plus(+) task, for instance, was expected from the strong emergent dimensions of the integral map types, however the best performer in this task was the bar chart, a configural solution. The X and Y tasks were hypothesized to be best supported by the separable combinations, but this materialized in only one instance (the choropleth with graduated symbols, in the General Y task). Additionally, participants did not rate separable or integral combinations as better at portraying individual distributions versus relational information. The inability of selective attention theory to describe all of the experimental results suggest that task-based map reading has fundamental perceptual and cognitive differences from interacting with the same sorts of symbols in an abstracted speeded classification setting. Future studies examining selective attention in a Cartographic setting should be mindful of these differences.

The eight map types showed frequent variations in reaction times across task,

whether that be performing better than the other maps in one of the eight tasks (cataloged in **Table 4.7**), or particular map types showing variation in performance between Elementary and General level tasks within a given information axis (**Table 4.8**). There were also statistically significant differences between the eight tasks in terms of their accuracy rates. Speaking broadly, variations in task performance across the map types were relatable to the unique perceptual properties of their specific visual cues. In one case, the spoke glyph, the lack of strong visual cues made it challenging for participants to retrieve information regardless of task. The other map types were generally successful in supporting the eight tasks overall, but showed differences in performance dependent on the information axis or the level of reading. The configural map types, for instance, demonstrated a good ability for viewers to extract information along the Minus(-) axis, but only when attending to individual symbols (Elementary tasks); when challenged to perform the same task at the General level, participants seemed unable to visually aggregate the symbols into a similarly intuitive cue. Variations in accuracy across the eight tasks had modest but statistically significant differences. It is possible that participants occasionally forgot which visual variable encoded which statistical attribute, leading to the greater number of errors in the X, Y, and Minus(-) tasks.

Performance across the different map types, in both accuracy and response time, did not significantly vary according to user expertise. It is possible that the expertise of the participants was not broad enough, with most being recruited from

within the same Geography Department. Alternatively, expertise in mapmaking and GIS may not provide the specific map literacy skills that would provide an advantage over laypersons in completing the specific information extraction tasks required in this study.

Finally, several of the experimental results indicate that although bivariate maps can provide sensible visual cues to higher-level information, these cues were not consistently noticed or leveraged by the map viewers. Specifically, the fact that 1) reaction time results among the Plus(+) and Minus(-) tasks did not frequently vary based on selectivity, 2) Experts and novices did not vary significantly in their reaction times, and 3) participants did not report a difference between separable/integral combinations as far as showing individual distributions vs relational information. Selective attention is helpful in examining the limits of human perception. However, in this particular experiment the limitations of the map appeared to be founded not on perception, but cognition. It is possible that instructing map readers on how to extract higher-level (Plus[+] and Minus[-] axis) information from the map would be a powerful means to enhance bivariate map reading. Similar map literacy practices already exist; the contour lines in a topographic map, for instance, portray the steepness of a slope (by the closeness of the lines) and its aspect (by determining the angle perpendicular to the contour lines), but this information is not readily apparent to a viewer unfamiliar with contour lines as a form of terrain representation. While it is outside the scope of this writing to suggest specific means to enhance

bivariate map literacy, several options could be feasible. Map designers could place explanatory notes on the map itself, cueing readers to the existence of the map's emergent dimensions and how they relate to higher-level information within the graphic. Alternatively, techniques for bivariate map literacy could be provided in a classroom setting, incorporated into any educational material on mapmaking, spatial analysis, or visual literacy.

5.2 Concluding Design Considerations

As examined throughout **Chapter 2**, there are numerous design considerations for bivariate maps: an attempt to formally catalog these design considerations is below.

1) Characteristics of the Mapped Phenomenon: These include the phenomena's dimensionality (point, line, or polygon), level of measurement (categorical, ordinal, numerical), data format (vector or raster), and continuity/abruptness (per MacEachren & DiBiase, 1991). These characteristics guide which visual variables, symbol dimensionalities, and overall representation techniques will be appropriate for visually representing each set of attributes, identical to how they guide representation choices in the production of a univariate map. **Tables 2.4 – 2.6**, for instance, were scoped to solutions appropriate for numerical data enumerated to polygon features.

2) Conceptual Relationships Between the Phenomenon: Though not

acknowledged into the empirical study, the conceptual relationship between the mapped phenomenon are one way in which selectivity can inform symbolization choice. It is unknown if following these guidelines would result in an appreciably improved performance in map reading (at the very least, no such result was directly visible in this study), but they relate clearly to the perceptual models from selective attention, and similar guidelines have been offered by Nelson (1999; 2000).

- *Separability* may be preferred when the mapped attributes are two independent variables that have incongruous scales (for instance, crime rates in arrests per capita vs. median income in dollars, or temperature in degrees vs. precipitation in inches). If the two variables have congruous scales, a configural combination may be more appropriate (see below).
- *Integrality* may be preferred when the information encoded along the emergent dimension (generally the Plus(+) axis) is a meaningful data variable unto itself. A cartographer, for instance, may define 'affluence' as a combination of property value and educational attainment. An integral visual combination would saliently demonstrate that the emergent information item is a product of the combined influence of those two constituent attributes.
- *Configurality* may be preferred when the mapped attributes are two variables that do have congruous scales (for instance, corn export in bushels vs. wheat export in bushels, or HIV rates in males vs. HIV rates in females). The most defining feature of a configural combination is its emergent feature in the

Minus(-) axis when the two attributes are 'in agreement' (e.g., the perfect rectangle formed when the two halves of a bar chart share the same attribute). Using information with congruous scales ensures the visual 'agreement' in the symbol represents a meaningful 'agreement' in the data attributes.

- *Asymmetry* may be preferred when one of the mapped attributes is more important to communicate than the other, or when the goal is to examine conditional relationships within the attributes (for instance, '*how does X vary when considering only the higher values of Y?*'). Value-by-alpha maps, for instance, were initially described as a tool for election mapping, using transparency to downplay the visual impact of states with low populations (Roth, Woodruff, & Johnson, 2010). The ability to attend to changes in hue diminishes as transparency increases, and this asymmetric inhibition is used to meaningfully highlight and dehighlight features on the map.

3) Context-Specific Information Communication Goals: These refer to any possible information representation goals (i.e., tasks) that the cartographer desires to emphasize or de-emphasize within the context of a specific project. These goals will inform any number of design decisions within the map, such as map type, classification scheme, symbol scaling, sequential vs. divergent color ramps, legend design, redundant encoding, and many more. Broadly speaking, mapmakers should be cognizant of the sorts of emergent dimensions and features created by their

symbolizations (as preliminarily examined in **Section 2.2.3**), and how the strength, nature, and interactions of these visual cues might serve to direct viewers' attention within the map. The findings of the empirical study provide a few possible ways in which choice of map type can enhance information communication goals. **Tables 4.6 and 4.7** can serve as a reference as to which map types better support specific tasks. The General Minus(-) task, for instance, was better supported by the choropleth with graduated symbol and bivariate choropleth map, but poorly supported by the two configural map types (bar chart and spoke glyph).

5.3 Future Directions

This empirical study, drawing from theory of selective attention, assumed that the major challenges of reading a bivariate map were perceptual: that is, the visual variables interact with each other in such a way as to hinder information extraction from the map, and choice of map type (including its selectivity) could serve to minimize visual interference and maximize the intuitive representation of the information (or, at least, certain aspects of the information). As described in **Section 5.1**, though, perceptual considerations did not account for all of the findings in the experiment: cognition appears to also be an important aspect of successful bivariate map reading. This was not anticipated in the experimental design, and should be considered as a limitation of the study. Other than the Likert scales, the information collected in the survey demonstrates very little about how the bivariate maps were

cognitively understood by the participants. Understanding the cognitive aspects of bivariate map reading provides an avenue for future research in bivariate Thematic Cartography. In order to understand why reaction times varied across task, for instance, it would have been beneficial to know the mental strategies the participants used to answer the questions. Did the participants begin by looking at the map, or at the question itself? How often, and in what contexts, did participants have to re-check the legend? Did they answer the General-level questions by attending to individual map features, or by trying to visually aggregate them? There are several methods that could better investigate such cognitive-based questions, such as eye tracking, focus groups, "think-aloud" experiments (Pickle 2003), or by giving participants more open-ended information-seeking challenges (rather than the highly specific information retrieval tasks used in this study).

Similarly, this writing attempted to identify and discuss the emergent dimensions and features of various bivariate map types; dimensions such as the 'leftedness/rightedness' of a bar chart or split proportional symbol, or the directionality (vertical/horizontal) of a rectangle map. The understanding of these various emergent visual dimensions is significantly less than the understanding of the more fundamental visual variables. Since these emergent dimensions do serve to encode information in a multivariate map, it would be enlightening to examine how these dimensions are perceived and their appropriateness for encoding different varieties of information.

Glossary

Asymmetrical: A condition of selectivity marked by conditional interaction effects between the constituent visual variables. Asymmetry generally occurs when one visual variable acts as a stronger cue than the other.

Bar Chart: A bivariate map solution using bars of different sizes to represent each data variable. Bar charts are a *configural* solution using a *size/size* and *point/point* construction.

Baseline Task: A task within speeded classification where a participant is provided with one symbol and must locate an identical symbol within a series of others. The other symbols vary in only one visual dimension.

Bivariate Choropleth: A bivariate map solution using a two-dimensional color ramp. Bivariate choropleths are (generally) an *integral* solution using *color/color* and *polygon/polygon* construction.

Bivariate Map: A thematic map that visually represents two data variables.

Configural: A condition of selectivity marked by a relatively strong emergent feature, and a fair-to-good ability to visually attend to both the emergent properties of the symbol as well their constituent components. Configurality generally occurs when a symbol set employs two identical visual variables, such as *bar charts*, *spoke glyphs*, and *split symbols*.

Compare: A map-reading *task* wherein the user compares the values of two map features. Contrast with *identify*.

Condensation Task: A task within speeded classification where a participant is provided with one symbol and must locate an identical symbol within a series of others. The other symbols vary in two visual dimensions, and the viewer must successfully attend to both.

Condition of Selectivity: See **Selectivity**.

Choropleth with Graduated Symbols: A bivariate map solution consisting of proportional symbols overlaid onto a choropleth. Choropleths with graduated

symbols are a highly *separable* solution using a *color/size* and *point/polygon* construction.

Dimensionality: How many spatial dimensions a symbol is composed of: can be points (0D), lines (1D), polygons (2D), or volumes (3D).

Elementary Map Reading Task: A map reading task that requires attending to only one map feature at a time. Contrast with *general* map reading tasks.

Emergent Feature: A visual feature created by combining two particular visual variables within particular circumstances. Width and height, for instance, form the emergent feature of a perfect square, but only if the width and height are identical. This writing distinguishes between emergent features and *emergent dimensions*; emergent features are a binary (an individual symbol either exhibits the emergent feature, or it does not), whereas emergent features are orderable (individual symbols exhibit the emergent dimension to varying degrees).

Emergent Dimension: An attendable visual dimension created by the combined influence of two visual variables. Height and width, for instance, form the emergent dimension of area.

Filtering Task: A task within speeded classification where a participant is provided with one symbol and must locate an identical symbol within a series of others. The other symbols vary in two visual dimensions, and the viewer must attend to one while ignoring the confounding influence of the other.

General Map Reading Task: A map reading task that requires attending to multiple, usually contiguous, map features simultaneously. Contrast with *elementary* map reading tasks.

Identify: A map-reading *task* wherein the user identifies the value of a single map feature.

Information Axis: An axis onto which meaningful variations of data exist across. In a bivariate map, the key information axes are the two constituent data variables (X and Y), their positive association (Plus[+]), and their negative association (Minus[-]).

Integral: A *condition of selectivity* marked by reduced ability to attend to the constituent visual variables, instead focusing on their emergent dimension.

Map Type: Competing forms of visual data representation within thematic mapping.

Map Literacy: A generalized set of knowledge of how to accurately and efficiently retrieve information from a map.

Map Reading Tasks: see *tasks*

Minus(-) Axis: The *data axis* representing the negative association between the data variables: that is, whether $X > Y$ or $Y > X$.

Multivariate: A map representing three or more data variables.

Orthogonal Information Axis: Data axes representing relational information about two constituent data variables (the plus(+) axis and minus(-) axis).

Plus(+) Axis: The *information axis* representing the positive association between the data variables: that is, whether **both are low** or **both are high**.

Point/Point: A symbol construction wherein both visual variables are applied to a point symbol, with no statistical information encoded in the 'base map'.

Point/Polygon: A symbol construction wherein a point symbol is superimposed onto a polygon symbol.

Polygon/Polygon: A symbol construction wherein visual variables are applied to the enumeration units within the 'base map'

Rectangle Map: A map type composed of symbols that vary by height and width. An *integral* solution using a *height/width* and *point/point* construction.

Region: In this study, region was an operational definition used to describe a collection of four contiguous *units* on the map.

Redundancy: The use of two or more visual variables to encode identical information.

Redundancy Task: A task within speeded classification where a participant is provided with one symbol and must locate an identical symbol within a series of others. The other symbols vary in two visual dimensions, but the viewer need only to attend to one of them.

Selective Attention: 1) the human perceptual ability to, when presented with a complex visual scene, attend to specific visual channels while ignoring the others. 2) a subdiscipline of Psychology that investigates this phenomena.

Selectivity: Categories that broadly organize the selective attentive characteristics of a visual combination. The four selectivities treated here are *separable*, *integral*, *configural*, and *asymmetrical*.

Separable: A condition of selectivity marked by an ability to easily attend to the constituent visual variables, at the expense of creating strong emergent dimensions.

Shaded Cartogram: A map type composed of changes in size to the enumeration units, and applying different colors to those units. A separable solution using a *size/value* and *polygon/polygon* construction.

Speeded Classification: A methodology within Selective Attention research that asks participants to match a given symbol to an identical one within a set of several symbols, with varying forms of visual interference.

Spoke Glyph: A map type composed of bars that vary in orientation. A configural solution using an *orientation/orientation* and *point/point* construction.

Tasks: Specific activities the map reader can engage in to extract information from the map.

Thematic Map: Traditionally, a map designed for a one-off purpose. Now, usually used to refer to maps designed to convey statistical data.

Unit: In this study, unit was an operational definition used to describe a single enumeration unit on the map.

Univariate Map: A thematic map portraying a single data variable.

Value-by-alpha: A map type composed of varying the transparency (alpha) and hue of the map's enumeration units. An asymmetrical solution using a *transparency/hue* and *polygon/polygon* construction.

Visual Aggregation: A perceptual/cognitive process wherein the viewer must sum the influence of multiple visual features, and decide upon their overall, representative value. Fundamental to *general* level map reading tasks.

Visual Complexity: The amount of interfering graphical dimensions within a map.

Visual Variables: The low level graphical dimensions within an image.

X: A data axis representing one of the base-level data variables on the map.

Y: A data axis representing one of the base-level data variables on the map.

References

- Amar, R., Eagan, J., & Stasko, J. (2005). Low-level components of analytic activity in information visualization. *Information Visualization*, Minneapolis, MN. IEEE.
- Andrienko, N., Andrienko, G. & Gatalsky, P. (2003) Exploratory spatio-temporal visualization: An analytical review. *Journal of Visual Languages and Computing*, 14, 503-541.
- Antes, J.R., & Chang, K. (1990). An Empirical Analysis of the Design Principles for Quantitative and Qualitative Area Symbols. *Cartography and Geographic Information Systems*, 17(4), 271-277
- Beard, K., & Mackaness, W. (1993). Visual access to data quality in geographic information systems. *Cartographica*, 30(2), 37–45.
- Bertin, J. (1967|1983). *Semiology of graphics*. Madison, Wis.: University of Wisconsin Press.
- Blok, C., Kobben, B., Cheng, T. & Kuterema, A.A. (1999) Visualization in relationships between spatial patterns in time by cartographic animation. *Cartography and Geographic Information Science*, 26, 139-151.
- Brewer, C. (1989). Color Chart Use in Map Design. *Cartographic Perspectives*, 4, 3–10.
- Brewer, C. A., MacEachren, A., Pickle, L., & Hermann, D. (1997). Mapping Mortality: Evaluating Color Schemes for Choropleth Maps. *Annals of the Association of American Geographers*, 87(3), 411–438.
- Brewer, C., & Campbell, A. (1998). Beyond Graduated Circles: Varied Point Symbols for Representing Quantitative Data on Maps. *Cartographic Perspectives*, 29, 6–25.
- Brewer, C., & Olson, J. (1997). An Evaluation of Color Selections to Accomodate Map Users with Color-Vision Impairments. *Annals of the Association of American Geographers*, 87(1), 103–134.
- Brewer, C.A., and L.W. Pickle. 2002. Evaluation of methods for classifying

- epidemiological data on choropleth maps in series. *Annals of the Association of American Geographers* 82(4): 662-81.
- Caivano, J. L. (1990). Visual texture as a semiotic system. *Semiotica*, 80, 2–4.
- Carswell, C. M., & Wickens, C. D. (1990). The perceptual interaction of graphical attributes: Configurality, stimulus homogeneity, and object integration. *Perception & Psychophysics*, 47(2), 157–168. doi:10.3758/BF03205980
- Cliburn, D. C., Feddema, J. J., Miller, J. R., & Slocum, T. A. (2002). Design and evaluation of a decision support system in a water balance application. *Computers & Graphics*, 26(6), 931-949.
- Crampton, J. W. (2002). Interactivity types in geographic visualization. *Cartography and Geographic Information Science*, 29(2), 85-98.
- Dent, B. D., Torguson, J., & Hodler, T. W. (2009). *Cartography: thematic map design*. New York: McGraw-Hill Higher Education.
- Dunn, R. (1989). A dynamic approach to two-variable color mapping. *The American Statistician*, 43(4), 245-252.
- Evans, B. J. (1997). Dynamic display of spatial data-reliability: Does it benefit the map user?. *Computers & Geosciences*, 23(4), 409-422.
- Fisher, H. T. (1982). *Mapping information: the graphic display of quantitative information*. Cambridge, MA: Abt Books.
- Flannery, J. (1971). The Relative Effectiveness of Some Common Graduated Point Symbols in the Presentation of Quantitative Data. *Canadian Cartographer*, 8(2), 96–109.
- Gardner, Steven D. (2005). *Evaluation of the Colorbrewer Color Schemes for Accommodation of Map Readers with Impaired Color Vision*. (Master's Thesis). The Pennsylvania State University, University Park, Penn.
- Gale, N., and Halperin, W.C. (1982) A case for better graphics: The unclassed choropleth map. *The American Statistician* 36(4), 330-336.
- Harrower, M. (2007). The cognitive limits of animated maps. *Cartographica: The International Journal for Geographic Information and Geovisualization*, 42(4),

349-357.

- Hope, S., & Hunter, G. J. (2007). Testing the Effects of Thematic Uncertainty on Spatial Decision-making. *Cartography and Geographic Information Science*, 34(3), 199–214. doi:10.1559/152304007781697884
- Huffman, D.P. (2010). *On Varying the Detectability of Symbols to Encode Data Uncertainty* (Master's Thesis). University of Wisconsin - Madison, Madison, Wis.
- Jenks, G., & Caspall, F. (1971). Error on choroplethic maps: Definition, measurement, reductions. *Annals of the Association of American Geographers*, 61(2), 217–244.
- Jenny, B. & Kelso, N.V. (2007). Color Design for the Color Vision Impaired. *Cartographic Perspectives*, 58(3), 61-67.
- Johnson, Z. F. (2008). *Cartograms for Political Cartography: A Question of Design* (Master's Thesis). University of Wisconsin - Madison, Madison, Wis.
- Kobus, D. A., Proctor, S., & Holste, S. (2001). Effects of experience and uncertainty during dynamic decision making. *International Journal of Industrial Ergonomics*, 28(5), 275–290. doi:10.1016/S0169-8141(01)00022-1
- Krygier, J., & Wood, D. (2011). *Making maps: a visual guide to map design for GIS*. New York: Guilford Press.
- Leitner, M., & Butterfield, B. . (1993). Guidelines for the Display of Attribute Uncertainty. *Cartography and Geographic Information Science*, 27(1), 55–62.
- MacEachren, A. (1982a). The Role of Complexity and Symbolization Method in Thematic Map Effectiveness. *Annals of the Association of American Geographers*, 72(4), 495–513.
- MacEachren, Alan M. (1982b). Map Complexity: Comparison and Measurement. *Cartography and Geographic Information Science*, 9(1), 31–46. doi:10.1559/152304082783948286
- MacEachren, A., & Dibiase, D. (1991). Animated Maps of Aggregated Data: Conceptual and Practical Problems. *Cartography and Geographic Information Science*, 18(4), 221–119.
- MacEachren, A.M. (1992). Visualizing uncertain information. *Cartographic*

- Perspectives*, 13(Fall), 10–19.
- McGranaghan, M. (1993). A Cartographic View of Data Quality Visualization. *Cartographica*, 30(2 & 3), 8–19.
- MacEachren, Alan M. (1995). *How maps work: representation, visualization, and design*. New York: Guilford Press.
- McMaster, R., & McMaster, S. (2002). A History of Twentieth-Century American Academic Cartography. *Cartography and Geographic Information Science*, 29(3), 305–321. doi:10.1559/152304002782008486
- Monmonier, M. S. (1999). *Maps with the news: the development of American journalistic cartography*. Chicago: University of Chicago Press.
- Montello, D. R. (2002). Cognitive Map-Design Research in the Twentieth Century: Theoretical and Empirical Approaches. *Cartography and Geographic Information Science*, 29(3), 283–304. doi:10.1559/152304002782008503
- Morrison, J. L. (1974). A Theoretical Framework for Cartographic Generalization with the Emphasis on the Process of Symbolization. *International Yearbook of Cartography*, 14, 115–127.
- Nelson, E. (2000). The Impact of Bivariate Symbol Design on Task Performance in a Map Setting. *Cartographica*, 37(4), 61–78.
- Nelson, E., & Edwards, L. (2001). Visualizing Data Uncertainty: A Case Study using Graduated Symbol Maps. *Cartographic Perspectives*, 38, 19–36.
- Nelson, Elizabeth. (1999). Using Selective Attention Theory to Design Bivariate Point Symbols. *Cartographic Perspectives*, 32(Winter), 6–29.
- Olson, J. M. (1981). Spectrally Encoded Two-Variable Maps. *Annals of the Association of American Geographers*, 71(2), 259-276.
- Pickle, L.W. (2003) Usability testing of map designs. *Proceedings of Symposium on the Interface of Computing Science and Statistics*. 2003.
- Pomerantz, J. R., & Schweitberg, S. D. (1975). Grouping by proximity: Selective attention measures. *Attention, Perception, & Psychophysics*, 18(5), 355-361.

- Robinson, A. H. (1995). *Elements of cartography*. New York: Wiley.
- Roth, R. E. (2009). The Impact of User Expertise on Geographic Risk Assessment under Uncertain Conditions. *Cartography and Geographic Information Science*, 36(1), 29–43. doi:10.1559/152304009787340160
- Roth, R. E., Woodruff, A. W., & Johnson, Z. F. (2010). Value-by-alpha Maps: An Alternative Technique to the Cartogram. *Cartographic Journal, The*, 47(2), 130–140. doi:10.1179/000870409X12488753453372
- Roth, R.E. (2011). *Interacting With Maps: The Science and Practice of Cartographic Interaction*. (Doctoral Dissertation). The Pennsylvania State University, University Park, PA.
- Shortridge, B. G. (1982). Stimulus Processing Models from Psychology: Can We Use Them in Cartography? *Cartography and Geographic Information Science*, 9(2), 155–167. doi:10.1559/152304082783948501
- Slocum, T. A., McMaster, R.B., Kessler, F.C., Howard, H.H. (2003). *Thematic cartography and geographic visualization*. Upper Saddle River, N.J.; London: Prentice Hall.
- Slocum, T. A., Sluter, R. S., Kessler, F. C., & Yoder, S. C. (2004). A qualitative evaluation of MapTime, a program for exploring spatiotemporal point data. *Cartographica: The International Journal for Geographic Information and Geovisualization*, 39(3), 43-68.
- Stefan, H., René, S., Marianne, R., & Lorenz, H. (2007). Multivariate Mapping in High Quality Atlases. *Proc. of the 23th Int. Conference of the ICA*. Moskow, Russia.
- Trumbo, B. E. (1981). A theory for coloring bivariate statistical maps. *The American Statistician*, 35(4), 220-226.
- Tyner, J. A. (2010). *Principles of map design*. New York: Guilford Press.
- Wehrend, S. & Lewis, C. (1990) A problem-oriented classification of visualization techniques. *Visualization*. San Francisco, CA, IEEE.
- Yi, J. S., Kang, Y., Stasko, J. T., & Jacko, J. A. (2007). Toward a deeper understanding of the role of interaction in information visualization.

Visualization and Computer Graphics, IEEE Transactions on, 13(6), 1224-1231.

Zhou, M. X., & Feiner, S. K. (1998). Visual task characterization for automated visual discourse synthesis. *Proceedings of the SIGCHI conference on Human factors in computing systems* (392-399). ACM Press/Addison-Wesley Publishing Co.

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